

MODUL MATEMATIK TAMBAHAN



PROJEK
JAWAB UNTUK JAYA
(JUJ)2012

Wahana Kecemerlangan **SPM** Negeri Pahang Darul Makmur

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PENDAHULUAN

Modul Matematik Tambahan dalam Projek Jawab Untuk Jaya(JUU) 2012 Negeri Pahang disediakan khusus untuk menambahkan informasi serta latihan kepada pelajar-pelajar disamping Praktis Bestari. Pada tahun 2012 tajuk yang dipilih adalah *Statistics* dan *Trigonometric Functions*. Saban tahun panel JUU Matematik Tambahan telah menyediakan sekurang-kurangnya dua topik untuk dijadikan Modul dalam JUU. Topik *Statistics* dan *Trigonometric Functions* merupakan tajuk yang penting untuk dikuasai terutamanya dalam Kertas 2 kerana kedua-dua topik ini merupakan tajuk yang popular dan kekerapan keluar adalah sangat tinggi. Penguasaan pelajar dalam tajuk ini amat dituntut kerana peruntukan markahnya adalah tinggi. Turut disertakan sekali dalam modul ini adalah analisis kertas soalan Matematik Tambahan SPM bermula tahun 2003 sehingga 2011. Analisis ini boleh membantu pelajar mahupun guru mengkaji pecahan dan bilangan soalan bagi tahun-tahun terdahulu. Melalui analisis ini juga pelajar dan guru boleh merancang ulangkaji dan latih tubi dengan lebih tersusun dan berfokus. Diharapkan dengan adanya modul ini, kemahiran para pelajar terutamanya dalam topik *Statistics* dan *Trigonometric Functions* dapat dipertingkatkan serta meningkatkan lagi keyakinan bagi menghadapi SPM 2012.

ANALISIS KERTAS SOALAN SPM (2003-2011)

PAPER 1			YEAR								
Form	Chapter		03	04	05	06	07	08	09	10	11
4	1	Functions	2	3	3	2	3	3	3	3	3
	2	Quadratic Equations	1	1	2	1	1	1	1	1	1
	3	Quadratic Functions	1	2	1	1	2	2	2	2	2
	4	Simultaneous Equations									
	5	Indices and Logarithms	2	2	3	3	2	2	2	2	2
	6	Coordinate Geometry	2	2	1	1	2	2	1	2	1
	7	Statistics			1	1	1	1	1	1	1
	8	Circular Measure	1	1	1	1	1	1	1	1	1
	9	Differentiation	2	2	2	3	2	2	3/2	2	1
	10	Solution of Triangle									
	11	Index Number									
5	1	Progressions	2	4	3	2	3	3	3	3	3
	2	Linear Law	1	1	1	1	1	1		1	1
	3	Integration	2	1	1	2	1	1	5/2	1	2
	4	Vectors	3	2	2	2	2	2	2	2	2
	5	Trigonometric Functions	2	1	1	1	1	1	2	1	2
	6	Permutations / Combinations	2	1	1	1	1	1	1	1	1
	7	Probability		1	1	1	1	1	1	1	1
	8	Probability Distributions	2	1	1	1	1	1	1	1	1
	9	Motion Along A Straight Line									
	10	Linear Programming									
		Total	25	25	25	25	25	25	25	25	25



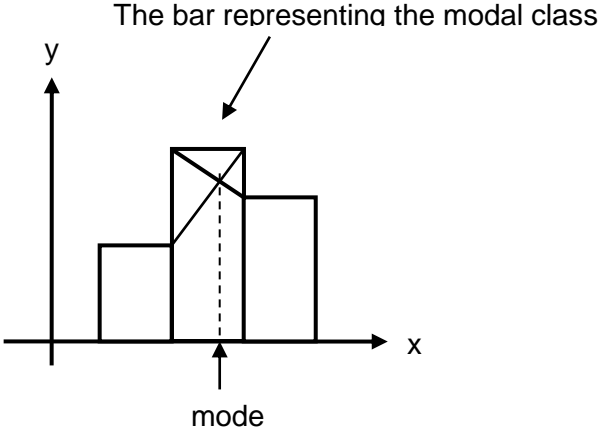
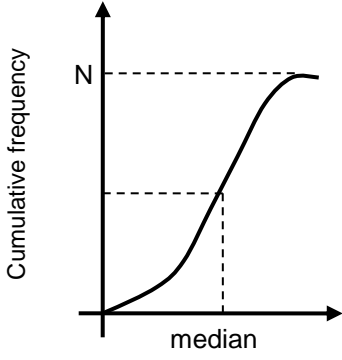
PAPER 2		SECTION / YEAR																			
Form	Chapter	Section A									Section B										
		03	04	05	06	07	08	09	10	11	09	04	05	06	07	08	09	10	11		
4	1	Functions				1					1										
	2	Quadratic Equations								1/2											
	3	Quadratic Functions	1						1	1/2											
	4	Simultaneous Equations	1	1	1	1	1	1	1	1											
	5	Indices and Logarithms									1										
	6	Coordinate Geometry					1				1	1	1		1	1		1	1		
	7	Statistics	1	1	1	1	1	1			1	1									
	8	Circular Measure	1												1	1	1	1	1	1	
	9	Differentiation	1	1/2	1/2			2/3			1				1/2	1/2	1/3		1/3	1/3	1
	10	Solution of Triangle																			
	11	Index Number																			
5	1	Progressions		1	1	1	1	1	1	1											
	2	Linear Law												1	1	1	1	1	1	1	
	3	Integration		1/2	1/2			1/3			1			1/2	1/2	2/3	1	2/3	2/3	1	1
	4	Vectors	1			1	1			1	1				1					1	1
	5	Trigonometric Functions		1	1	1	1	1	1	1	1			1							
	6	Permutations / Combinations																			
	7	Probability																			
	8	Probability Distributions													1	1	1	1	1	1	1
	9	Motion Along A Straight Line																			
	10	Linear Programming																			
		Total	6	6	6	6	6	6	6	6	6	6	6	5	5	5	5	5	5	5	5

PAPER 2		SECTION / YEAR									
Form	Chapter	Section C									
		09	04	05	06	07	08	09	10	11	
4	1	Functions									
	2	Quadratic Equations									
	3	Quadratic Functions									
	4	Simultaneous Equations									
	5	Indices and Logarithms									
	6	Coordinate Geometry									
	7	Statistics									
	8	Circular Measure									
	9	Differentiation									
	10	Solution of Triangle	1	1	1	1	1	1	1	1	1
	11	Index Number	1	1	1	1	1	1	1	1	1
5	1	Progressions									
	2	Linear Law									
	3	Integration									
	4	Vectors									
	5	Trigonometric Functions									
	6	Permutations / Combinations									
	7	Probability									
	8	Probability Distributions									
	9	Motion Along A Straight Line	1	1	1	1	1	1	1	1	1
	10	Linear Programming	1	1	1	1	1	1	1	1	1
		Total	4	4	4	4	4	4	4	4	4

STATISTICS

FACTS AND FORMULAE

Measures of Central Tendency

	Ungrouped Data	Grouped Data
Mean	$\bar{x} = \frac{\sum x}{N}$ <p>$\sum x$ = Sum of all the values N = Number of data</p>	$\bar{x} = \frac{\sum fx}{\sum f}$ <p>x = midpoint of class f = class frequency</p>
Mode	Data that occurs most frequently	<p>Histogram</p> 
Median	The middle value in a set of data which is arranged in ascending or descending order	<p>Ogive</p>  $\text{Median, } m = L + \left(\frac{\frac{1}{2}N - F}{f_m} \right) C$ <p>L = Lower boundary of median class N = Total frequency C = Size of median class f_m = Frequency of median class F = Cumulative frequency before class median</p>

Measures of Dispersion

	Ungrouped Data	Grouped Data
Range	Largest Value/Data – Smallest Value/Data	Largest Class Mark/Mid Point – Smallest Class Mark/Mid Point
Interquartile range	<p>Third Quartile – First Quartile</p> $Q_3 - Q_1$	$Q_3 - Q_1$ $Q_3 = L_3 + \left(\frac{\frac{3}{4}N - F_3}{f_{Q_3}} \right) C$ $Q_1 = L_1 + \left(\frac{\frac{1}{4}N - F_1}{f_{Q_1}} \right) C$ <p>L = Lower Boundary N = Total Frequency F = Cumulative Frequency before 1st or 3rd quartile class f = Frequency of the 1st or 3rd quartile class C = Size of the class interval</p>
Variance	$\sigma^2 = \frac{\sum (x - \bar{x})^2}{N}$ $\sigma^2 = \frac{\sum x^2}{N} - \bar{x}^2$	$\sigma^2 = \frac{\sum f(x - \bar{x})^2}{\sum f}$ $\sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2$
Standard Deviation	$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$ $\sigma = \sqrt{\frac{\sum x^2}{N} - (\bar{x})^2}$	$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$ $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$
	<p>f = frequency x = class midpoint \bar{x} = mean = $\frac{\sum x}{N}$</p>	<p>f = frequency x = class midpoint \bar{x} = mean = $\frac{\sum fx}{\sum f}$</p>

Effects on Measures of Central Tendency

	Mode	Median	Mean
Initial measure of central tendency	a	b	c
Each value increased by k	$a + k$	$b + k$	$c + k$
Each value decreased by k	$a - k$	$b - k$	$c - k$
Each value multiplied by k	ka	kb	kc
Each value divide by k	$\frac{a}{k}$	$\frac{b}{k}$	$\frac{c}{k}$
Exist an extreme value	No effect	No effect	Too small or too big to be a suitable measure of central tendency

Effects on Measures of Dispersion

	Range	Interquartile range	Variance	Standard Deviation
Initial measure of dispersion	R	r	σ^2	σ
Each value increased by k	R	r	σ^2	σ
Each value decreased by k	R	r	σ^2	σ
Each value multiplied by k	kR	kr	$k^2\sigma^2$	$k\sigma$
Each value divide by k	$\frac{R}{k}$	$\frac{r}{k}$	$\frac{\sigma^2}{k^2}$	$\frac{\sigma}{k}$
Exist an extreme value	Too big to be a suitable measure of dispersion	No effect	Too big to be a suitable measure of dispersion	

STATISTICS

- 1 The mean of a set of data $2k - 3, 8, k + 1$ is 7.
 Find
 - (a) The value of k
 - (b) The new mean if each of the data multiplied by 3. [3 marks]

- 2 The set of positive integers $y, 2, 11, 5, 7, x, 9$ has a mean 8 and median 9.
 Find the values of x and of y if $y > x$. [3 marks]

- 3 A set of 10 scores $x_1, x_2, x_3, \dots, x_9, x_{10}$ has mean 9 and standard deviation 4.
 Find
 - (a) $\sum x$ [3 marks]
 - (b) $\sum x^2$

- 4 A set of numbers is $4, 7$ and h . The variance of this set of integers is 14.
 Find the values of h . [3 marks]

- 5 The mean of a set of data $1, 10, m, 16$, and $3m$ is q . If each value in the set is decreased by 4, the new mean is $\frac{4q}{7}$.
 Find the value of
 - (a) q
 - (b) m [3 marks]

- 6 The standard deviation of a set of six numbers is $\sqrt{15}$. Given that the sum of square for the set of numbers is 144. Find the new mean when the a number 10 is added to this set. [3 marks]

- 7 A set of positive integers consists of $5, 6, k, 1, 7, 2, 2$.
 - (a) Find the value of k if the mean of the data is 4
 - (b) State the range of the values of k if the median of the data is k . [3 marks]

8 Table 8 shows marks obtained by a group of students in a mathematics test.

Marks	1 – 20	21 – 40	41 – 60	61 – 80
Number of students	6	9	13	12

Table 8

Without drawing an ogive, find the third quartile mark. [3 marks]

9 Table 9 shows the frequency distribution of ages of teachers.

Age (years)	28 – 32	33 – 37	38 – 42	43 – 47	48 – 52
Number of teachers	16	38	26	11	9

Table 9

Given the median of ages of teachers is $P = Q + \left[\frac{50 - K}{L} \right] 5$.

Find the values of P , Q , K and L .

[4 marks]

10 A group of 5 students has a sum of the squares of their marks is 1260 and a variance of 40.

Find

- (a) The mean's mark of the 5 students
- (b) The total mark of the 5 students

[3 marks]

11 A set of data has a mean of 9 and a variance of 3.2 . Each number in the set is multiplied by 2 and then 5 is added to it.

For this set of numbers, find

- (a) The mean
- (b) The variance

[4 marks]

12 Given that the mean of a set data 5 , 9 , 2 , 11 and x is 8.

Find

- (a) the value of x ,
- (b) the variance

[4 marks]

13 The mean of the set of data $2 - 5a$, $4a$, $3a$ and a^2 is 9.25 . Find the possible values of a.

[3 marks]

14 A set of six numbers has a mean of 20.

- (a) Find the sum of these six numbers
- (b) When a number y is removed from the set, the new mean is 19 .
Find the value of y.

[3 marks]

15 A set of data, Q , has the following information :

$$\bar{x} = 9, \sum x = 81, \sum x^2 = 2412$$

Calculate the standard deviation.

[3 marks]

16 A set of data consists of three numbers. The sum of this numbers is \sqrt{k} , the sum of the squares of the numbers is 27 and the standard deviation is 2h. Express k in terms of h.

[3 marks]

- 17 A set of data consists of 3, 1, 4, 9, 6, 9 and 12. [3 marks]
Find the interquartile range of the data.
- 18 The total of monthly allowance of 10 college students was RM 2000 and the sum of squares of monthly allowances was RM 418000. A student whose monthly allowance was RM 200 left the college. [3 marks]
Find the variance of remaining monthly allowances
- 19 A set of numbers arranged in descending order is 22, 18, x , y , 7, 5, 4, 2. [3 marks]
Given that mean and mode are 10 and 7 respectively.
Find the values of x and y .
- 20 A set of data has a mean of 9 and a variance of 12. Each number in the set is divided by 2 and then is subtracted by 3.
Find
(a) mean
(b) standard deviation
for this set of numbers.

PAPER 2 QUESTION

- 1 Table 11 shows the frequency distribution of the age of a group of tourists who visited a National Museum.

Age	Frequency
5 – 9	3
10 – 14	6
15 – 19	8
20 – 24	15
25 – 29	k
30 – 34	1

Table 11

- (a) It is given that the first quartile age of the distribution is 15.125
 Calculate the value of k . [3 marks]
- (b) Calculate the standard deviation of the data [4 marks]
- 2 Table 12 shows the frequency distribution of the height of a group of students.

Height/tinggi (cm)	Number of students Bilangan pelajar
110 - 119	8
120 – 129	19
130 - 139	13
140 – 149	6
150 - 159	4

Table 12

- (a) Without drawing an ogive, find the median of the height
- (b) Calculate the variance of the height.
- 3 Table 13 shows the distribution of marks of an Additional Mathematics test for a group of students.

Marks	0 – 9	10 – 19	20 - 29	30 – 39	40 – 49
Number of students	2	5	10	m	7

Table 13

- (a) Determine the maximum value of m such that the modal class is 20 – 29. [1 mark]
- (b) Given that $m = 16$, find
- (i) the mean
- (ii) the variance marks of the students. [5 marks]

4 Table 14 shows the distribution of scores obtained by a participant in a quiz.

Scores	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60
Number of participants	3	6	8	p	4	2

Table 14

- (a) Given that mean score of the distribution is 28.5 , [3 marks]
 calculate the value of p .
- (b) Without using an ogive, calculate the median mark. [3 marks]

5 Table 15 shows a distribution of scores for a group of 40 students in a Olimpiad quiz.

Score	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49
Number of students	3	8	h	k	7

Table 15

Given that the third quartile score is 36.5.

- (a) Find the values of h and of k . [7 marks]
 (b) Hence, state the modal class

6 Table 16 shows the frequency distribution of the masses of a group of students.

Mass (kg)	Number of students
41 – 45	2
46 – 50	6
51 – 55	15
56 – 60	12
61 – 65	5

Table 16

- (a) Using a scale of 2 cm to 5 kg on the horizontal axis and 2 cm to 2 students on the vertical axis, draw a histogram to represent the frequency distribution of the masses. [4 marks]
 Hence, find the modal mass.
- (b) Calculate the standard deviation of the distribution. [4 marks]

- 7 Table 17 (a) shows the cumulative frequency distribution for the scores of 32 students in a test.

Score	< 10	< 20	< 30	< 40	< 50
Number of students	5	9	21	29	32

Table 17(a)

- (a) Based on Table 17(a), copy and complete Table 17(b)

[1 mark]

Score	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49
Number of students					

Table 17(b)

- (b) Without drawing an ogive, find the interquartile range of the distribution.

[5 marks]

- 8 Table 18 shows the distribution of the scores of 40 students in a quiz.

Marks	Numbers of students
6 – 10	7
11 – 15	11
16 – 20	g
21 – 25	10
26 – 30	h

Table 18

- (a) Given that the median score is 16.75, find the value of g and h .

[4 marks]

- (b) Calculate the standard deviation of the distribution.

- (c) What is the standard deviation if the score of each student is multiplied by 3 and then increased by 2?

[3 marks]

[1 mark]

- 9 The mean of a set of numbers 3, 6, x , $x + 2$, 7, 11, 8, 9, is 7.

Find

- (a) The value of x .

[2 marks]

- (b) The standard deviation of the numbers.

[3 marks]

If each of the numbers above is multiplied by 2 and then 3 is added to it, Find

- (c) (i) the mean

- (ii) the variance of the new set of numbers.

[2 marks]

- 10 (a) A set of positive integers consists of 1, 4, and k . The variance of this set of integers is 6. Find the value of k .

[3 marks]

- (b) Given that set $Q = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$. The sum of the numbers is 160 and the sum of the squares of the numbers is 3480. Find the mean and the variance for the 8 numbers.

[3 marks]

- (c) When m is added to set Q , the mean increased by 2. Find

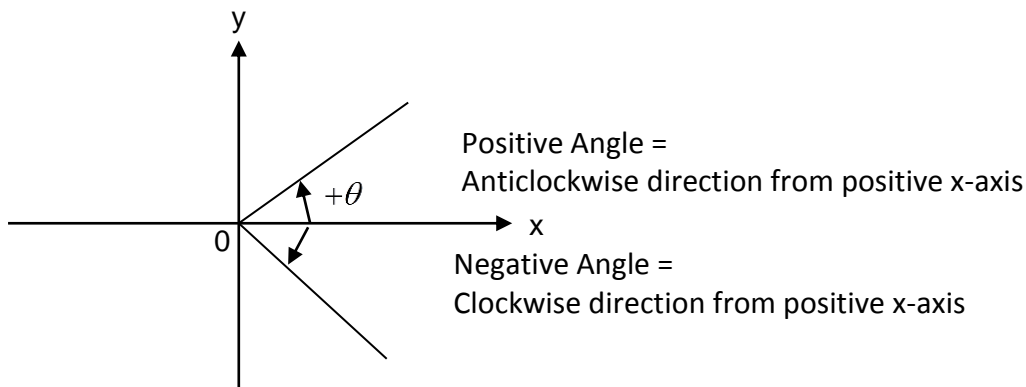
- (i) The value of m ,

- (ii) The standard deviation of the set of 9 numbers.

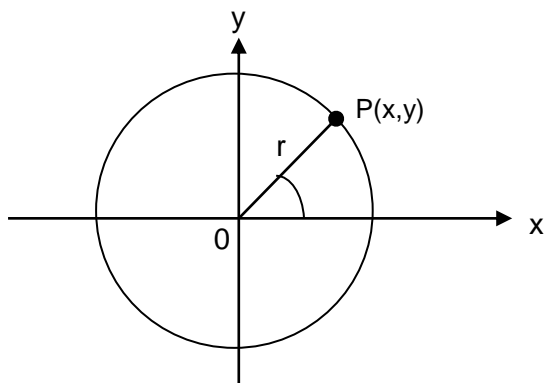
[4 marks]

TRIGONOMETRIC FUNCTIONS- FACTS AND FORMULAE

Positive and negative angles



Six Trigonometric Functions



$$\sin \theta = \frac{y - \text{coordinate}}{r}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{x - \text{coordinate}}{r}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\tan \theta = \frac{y - \text{coordinate}}{x - \text{coordinate}}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$= \frac{\sin \theta}{\cos \theta}$$

Complementary Angle for any acute angle θ

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\tan(90^\circ - \theta) = \cot \theta$$

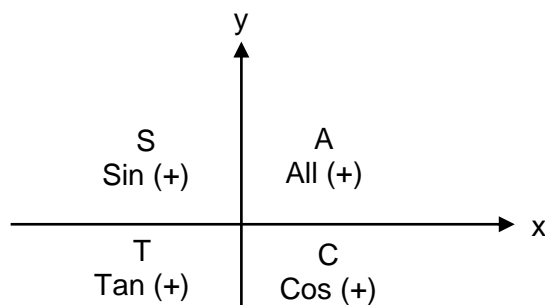
$$\sin(90^\circ - \theta) = \cos \theta$$

$$\operatorname{cosec}(90^\circ - \theta) = \sec \theta$$

$$\cot(90^\circ - \theta) = \tan \theta$$

$$\sec(90^\circ - \theta) = \operatorname{cosec} \theta$$

Sign of Trigonometric Functions

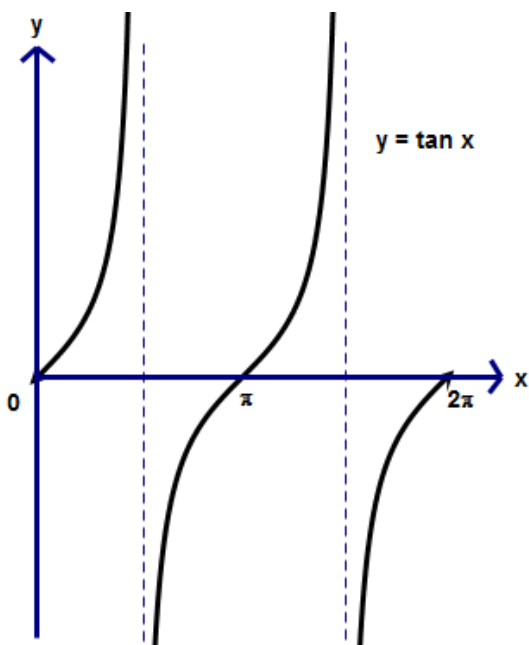
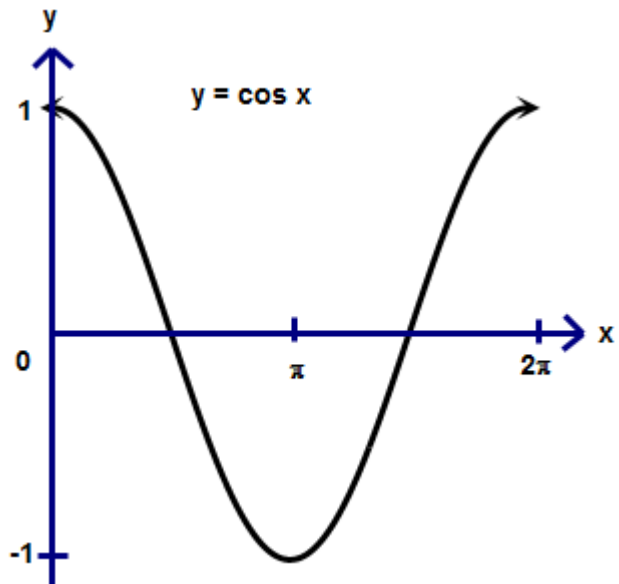
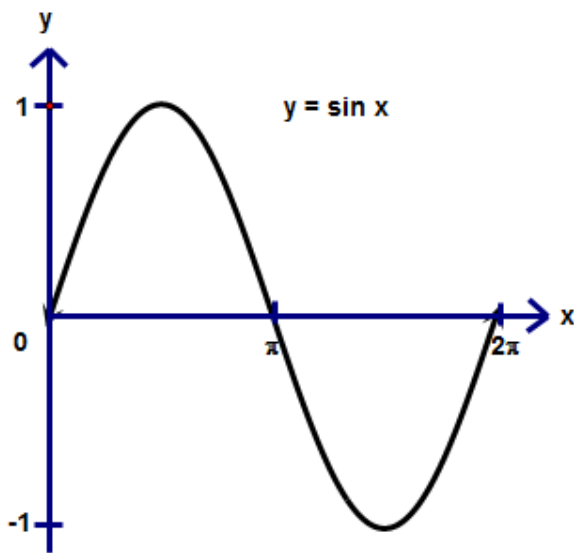


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Special Angles for Trigonometric Functions ($30^\circ, 45^\circ, 60^\circ$)

θ	30°	45°	60°
$\text{Sin } \theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\text{Cos } \theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\text{Tan } \theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Graph of Sine, Cosine and Tangent



Basic Identities

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

Addition Formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angle Formulae

$$\sin 2A = 2 \sin A \cos A$$

$$\sin 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Half Angle Formulae

$$\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$$

$$\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}$$

$$= 2 \cos^2 \frac{A}{2} - 1$$

$$= 1 - 2 \sin^2 \frac{A}{2}$$

$$\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$$

PAPER 1:

1. Solve the equation $2\sin x + \tan x = 0$ for $0^\circ \leq x \leq 360^\circ$
2. Solve the equation $\sin(2x + 45^\circ) = \cos 2x$ for $0^\circ \leq x \leq 360^\circ$
3. Solve the equation $\cos 2\theta = 2 - 3\sin \theta$ for $0^\circ \leq \theta \leq 360^\circ$
4. Solve the equation $3\sin^2 \theta + 5\cos \theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$
5. Solve the equation $3\cos 2\theta + \cos \theta + 1 = 0$ for $0^\circ \leq \theta \leq 270^\circ$
6. Solve the equation $\sin x - \sin 2x = 0$ for $0^\circ \leq x \leq 360^\circ$
7. Solve the equation $\cot 2\theta = -2.085$ for $0^\circ \leq \theta \leq 360^\circ$
8. Solve the equation $\sin(120^\circ - x) - \cos(x + 90^\circ) = 0$ for $0^\circ \leq x \leq 360^\circ$
9. Solve the equation $2\tan 2x - \cot x = 0$ for $0^\circ \leq x \leq 180^\circ$
10. Solve the trigonometric equation $\sin x \cos x = 0.5$ for $0^\circ \leq \theta \leq 360^\circ$
11. Given that $\cos A = -\frac{4}{5}$, where A is an obtuse angle. Without using the calculator find the value of $\sec\left(A + \frac{1}{2}\pi\right)$
12. It is given that $\cos \theta = m$, where θ is acute angle. Find:
 - (a) $\cot \theta$
 - (b) $\sin 2\theta$
 In term of m .

13. Given $\cos x = -p$ and x is an obtuse angle. Find the value of $\sin 2x$ in terms of p .
14. Given that $\sin x = -\frac{3}{5}$ and $\cos y = \frac{12}{13}$, where x and y in the same quadrant. Find the value of $\tan(x + y)$
15. Given that $\sin x = k$ and x is obtuse angle. Express in term of k .
- a) $\operatorname{cosec} x$
 b) $\tan(90^\circ - x)$
16. Given that $\cos 15^\circ = p$. Find the value
- a) $\cos 75^\circ$
 b) $\sin 30^\circ$
 In term of p .
17. It is given that $\tan x = \frac{4}{3}$ and $\sin y = \frac{5}{13}$, where x is an acute angle and y is an obtuse angle. Find
- a) $\tan y$
 b) $\cos(x + y)$
18. Given that $\sin x = -\frac{3}{5}$ and $90^\circ < x < 270^\circ$, find the value of $\sec 2x$.
19. Given that $\cos A = k$ such that A is reflex angle, express $\tan(360^\circ - A)$ in terms of k
20. Given that $\cos \theta = \frac{k}{2}$ and $0^\circ < \theta < 180^\circ$, find
- a) $\tan^2 \theta$
 b) $\cot \theta$

PAPER 2

1. a) Sketch the graph of $y = 1 + \sin 2x$ for $0 \leq x \leq 2\pi$.
 b) Hence, using the same axes, draw a suitable straight line to find the number of solution satisfying the equation $x + \frac{1}{2}\pi \sin 2x = \frac{1}{2}\pi$ for $0 \leq x \leq 2\pi$.
 State the number of solution.

2. a) Sketch the graph $y = \left| 2 \cos \frac{3}{2}x \right|$ for $0 \leq x \leq 2\pi$.
 b) Hence, using the same axes, sketch a suitable graph to find the number of solution $\left| \cos \frac{3}{2}x \right| - \frac{\pi}{x} = 0$ for $0 \leq x \leq 2\pi$.
 State the number of solutions.

3. a) Sketch the graph of $y = 1 - \sin 2x$ for $0 \leq x \leq 2\pi$.
 b) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $\frac{x}{\pi} = 1 + \sin 2x$ for $0 \leq x \leq 2\pi$.
 State the number of solution.

4. a) Sketch the graph $y = |\tan 2x| + 1$ for $0 \leq x \leq \pi$.
 b) Hence, using the same axes, sketch a suitable straight line to find the number of solution for the equation $|\tan 2x| = 2 - \frac{x}{\pi}$ for $0 \leq x \leq \pi$.
 State the number of solution.

5. a) Sketch the graph $y = |2 \cos 2x|$ for $0 \leq x \leq \frac{3}{2}\pi$.
- b) Hence, using the same axes, sketch a suitable straight line to find the number of solution for the equation $|4 \cos 2x| = \frac{2x}{\pi}$ for $0 \leq x \leq \frac{3}{2}\pi$.
- State the number of solution.

6. a) Prove that $\frac{4 \sin x}{\sqrt{1 + \tan^2 x}} = 2 \sin 2x$.

- b) Sketch the graph $y = |2 \sin 2x|$ for $0 \leq x \leq 2\pi$,

Hence, using the same axis, sketch a suitable line to find the number of solutions for

the equation $2\pi \left| \frac{4 \sin x}{\sqrt{1 + \tan^2 x}} \right| - x = 0$ for $0 \leq x \leq 2\pi$.

State the number of solution.



7. a) Sketch the graph $y = -2 \tan 2x$ for $0 \leq x \leq 2\pi$.

- b) Hence, using the same axes, sketch a suitable straight line to find the number of

solution for the equation $\pi \tan 2x = x - \pi$ for $0 \leq x \leq 2\pi$.

State the number of solution.

8. a) Prove that $\frac{2 + 2 \tan^2 \theta}{1 - \tan^2 \theta} = 2 \sec 2\theta$.

- b) Sketch the graph $y = 1 + 2 \cos x$ for $0 \leq x \leq 2\pi$.

- c) Hence, using the same axes, sketch a suitable straight line to find the number of

solution for the equation $\pi \cos x = \frac{x}{2} - \pi$ for $0 \leq x \leq 2\pi$

State the number of solution.

9. a) Sketch the graph $y = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$.

b) Hence, using the same axes, sketch a suitable straight line to find the number of

solution for the equation $\pi \cos \frac{x}{2} = x \sin \frac{x}{2}$ for $0 \leq x \leq 2\pi$

State the number of solution.

10. a) Prove that $\frac{\cos^2 x}{1 - \sin x} = 1 + \sin x$

b) Sketch the graph $y = 1 + \sin x$ for $0 \leq x \leq 2\pi$,

Hence, using the same axis, sketch a suitable line to find the number of solutions for

the equation $\frac{2 \cos^2 x}{1 - \sin^2 x} = \frac{x}{\pi}$ for $0 \leq x \leq 2\pi$.

State the number of solution.

ANSWERS-STATISTICS

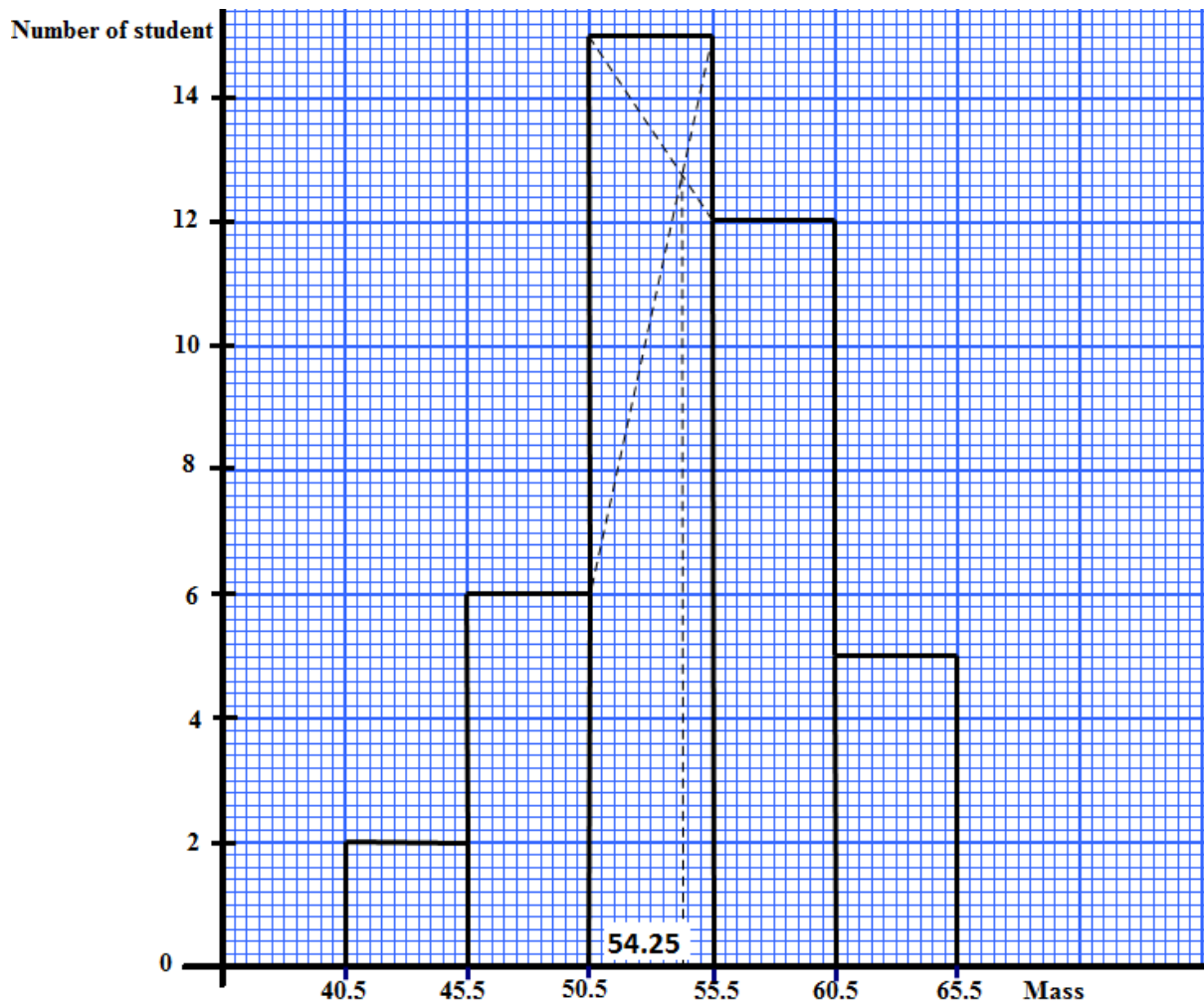
PAPER 1

NO	ANSWER	NO	ANSWER
1	(a) $k = 5$ (b) $\bar{x}_{new} = 21$	11	(a) 23 (b) 12.8
2	$x = 10, y = 12$	12	(a) $x = 13$ (b) $\sigma^2 = 16$
3	(a) $\sum x = 90$ (b) $\sum x^2 = 970$	13	$a = -7, 5$
4	$h = -2, 13$	14	(a) $\sum x = 120$ (b) $y = 25$
5	(a) $q = 9.333$ (b) $m = 4.916$	15	$\sigma = 13.67$
6	4	16	$k = 81 - 36h^2$
7	(a) $k = 5$ (b) $2 < k < 5$	17	6
8	63.83	18	RM 2000
9	$P = 36.97, Q = 32.5, K = 16, L = 38$	19	$x = 15, y = 7$
10	(a) 14.56 (b) 72.80	20	(a) 1.5 (b) 1.732

PAPER 2

NO	ANSWER
1	(a) $k = 7$ (b) 6.124
2	(a) 128.45 (b) 128.36
3	(a) $m = 9$ (b) (i) $\bar{x} = 29.75$ (ii) $\sigma^2 = 114.94$
4	(a) $p = 7$ (b) 28
5	(a) $k = 10, h = 12$ (b) 20 – 29
6	(a) 54.25 kg (b) 5.148 kg
7	(a) 5, 4, 12, 8, 3 (b) 16.25
8	(a) $g = 8, h = 4$ (b) 6.313 (c) 18.94
9	(a) $x = 5$ (b) 2.2913 (c) (i) 17 (ii) 21
10	(a) $k = 7$ (b) $\bar{x} = 20, \sigma^2 = 35$ (c) (i) $m = 38$ (ii) $\sigma = 7.944$

LAMPIRAN (SOALAN 6 (a))



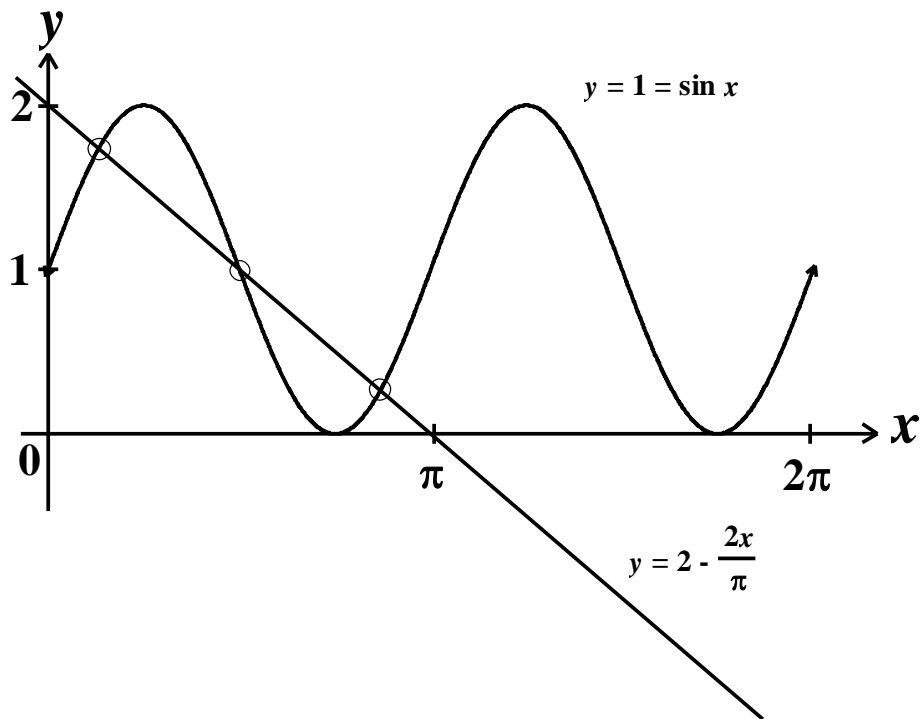
ANSWER – TRIGONOMETRIC FUNCTIONS

PAPER 1

No	Jawapan	No	Jawapan
1.	$x = 0^\circ, 120^\circ, 180^\circ, 240^\circ, 360^\circ$	11.	$-\frac{5}{3}$
2.	$x = 11.25^\circ, 101.25^\circ, 191.25^\circ, 281.25^\circ$	12.	a) $\frac{m}{\sqrt{1-m^2}}$ b) $2m\sqrt{1-m^2}$
3.	$\theta = 30^\circ, 90^\circ, 150^\circ$	13.	$-2p\sqrt{1-p^2}$
4.	$\theta = 109.47^\circ, 250.53^\circ$	14.	$-\frac{56}{33}$
5.	$\theta = 60^\circ, 131.81^\circ, 228.19^\circ$	15.	a) $\frac{1}{k}$ b) $-\frac{\sqrt{1-k^2}}{k}$
6.	$x = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$	16.	a) $\sqrt{1-p^2}$ b) $2p\sqrt{1-p^2}$
7.	$\theta = 77.19^\circ, 167.19^\circ, 257.19^\circ, 347.19^\circ$	17.	a) $-\frac{5}{12}$ b) $-\frac{56}{65}$
8.	$x = 150^\circ, 330^\circ$	18.	$\frac{25}{7}$
9.	$x = 24.09^\circ, 155.91^\circ$	19.	$-\frac{\sqrt{1-k^2}}{k}$
10.	$x = 45^\circ, 225^\circ$	20.	a) $\frac{4-k^2}{k^2}$ b) $\frac{k}{\sqrt{4-k^2}}$

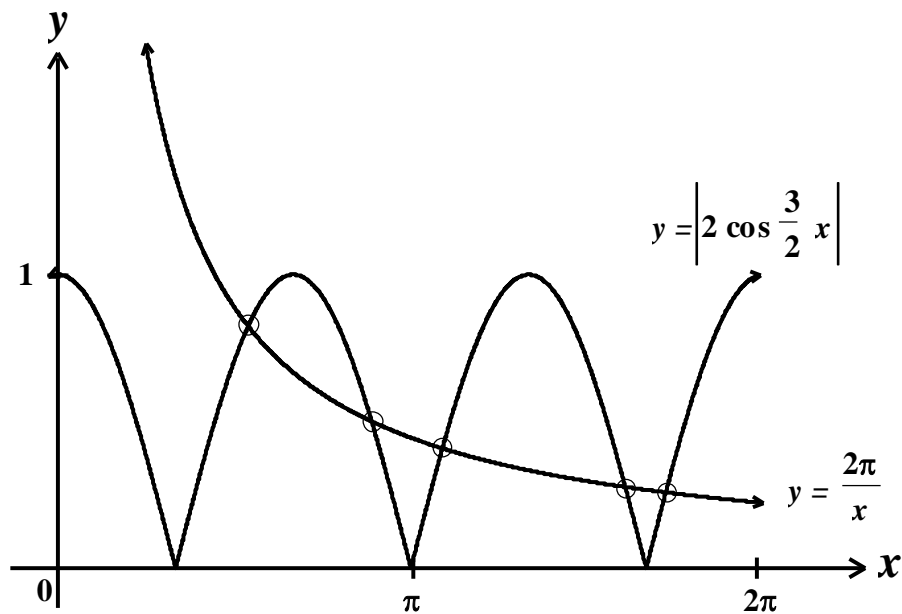
PAPER2

1. a)



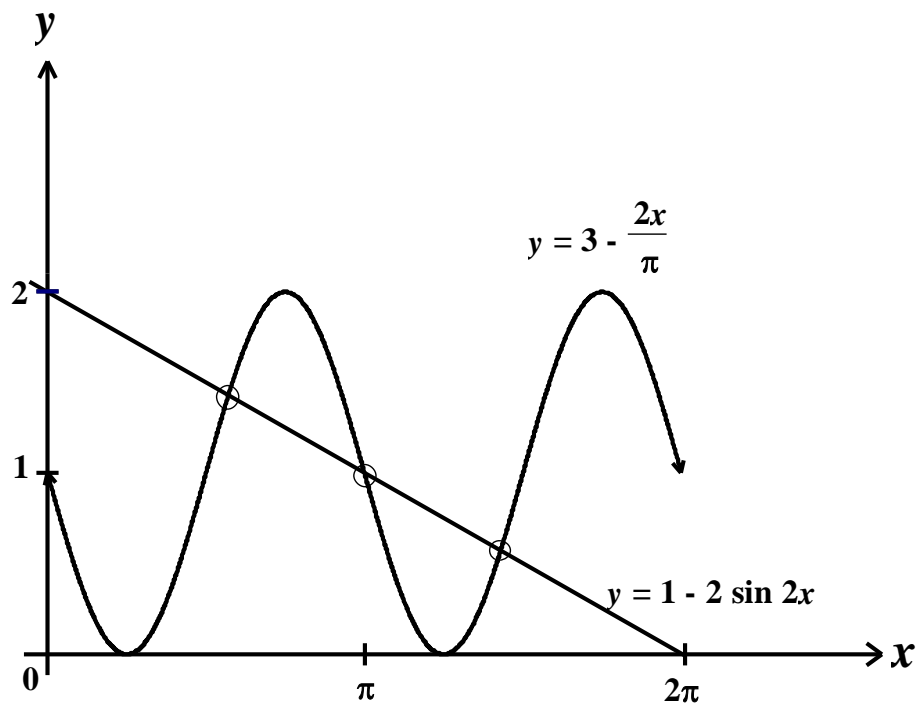
b) Draw a line $y = 2 - \frac{2x}{\pi}$. No of solution = 3

2. a)



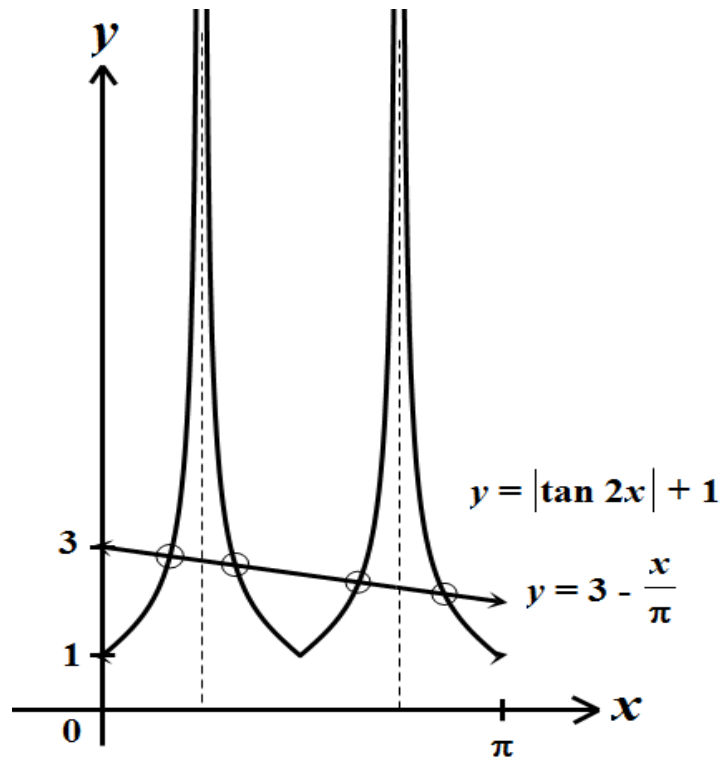
b) Draw a line $y = \frac{2\pi}{x}$. No of solution = 5

3. a)



b) Draw a line $y = 2 - \frac{x}{\pi}$. No of solution = 3

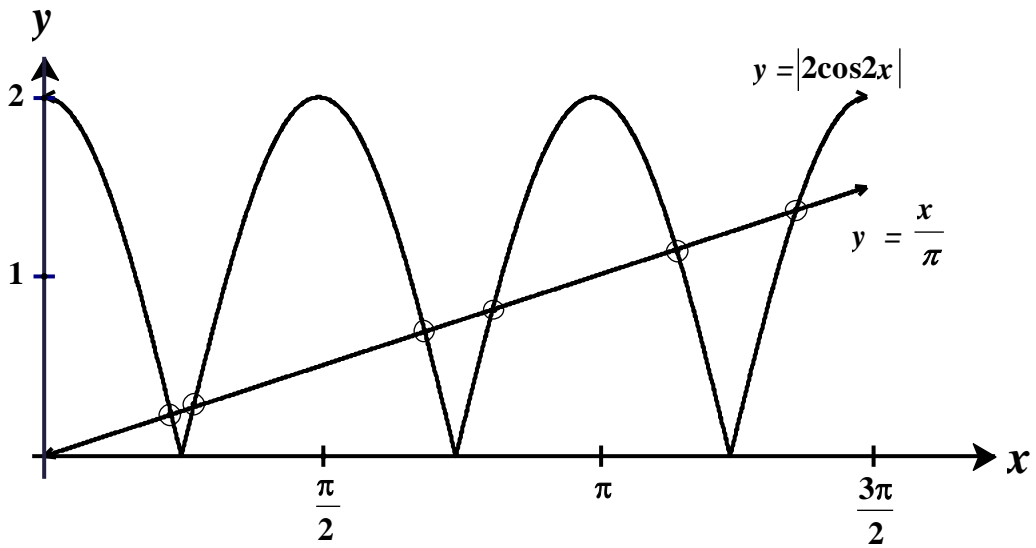
4. a)



b) Draw a line $y = 3 - \frac{x}{\pi}$. No of solution = 4

5.

a)



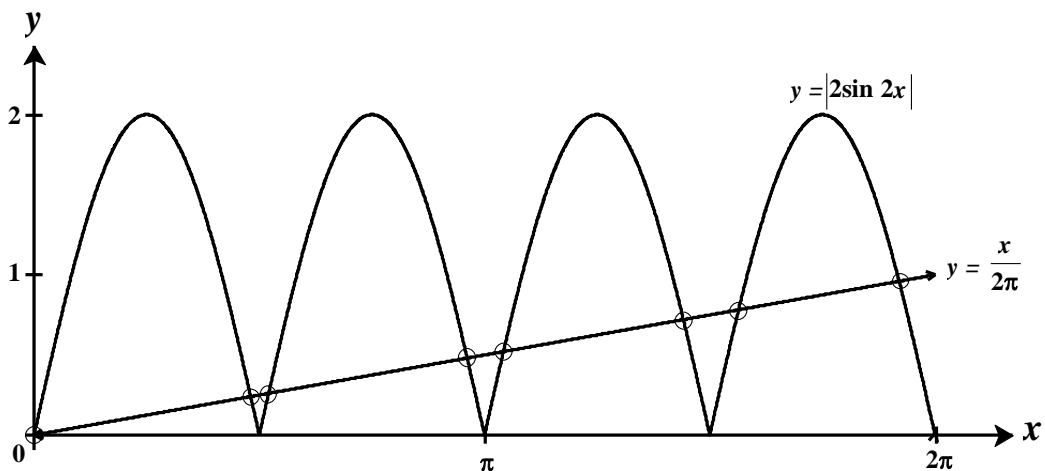
b) Draw a line $y = \frac{x}{\pi}$. No of solution = 6

6.

a) left side $\frac{4 \sin x}{\sqrt{1 + \tan^2 x}}$

$$\begin{aligned} &= \frac{4 \sin x}{\sec x} \\ &= \frac{4 \sin x}{\frac{1}{\cos x}} \\ &= 4 \sin x \cos x \\ &= 2(2 \sin x \cos x) \\ &= 2 \sin 2x \end{aligned}$$

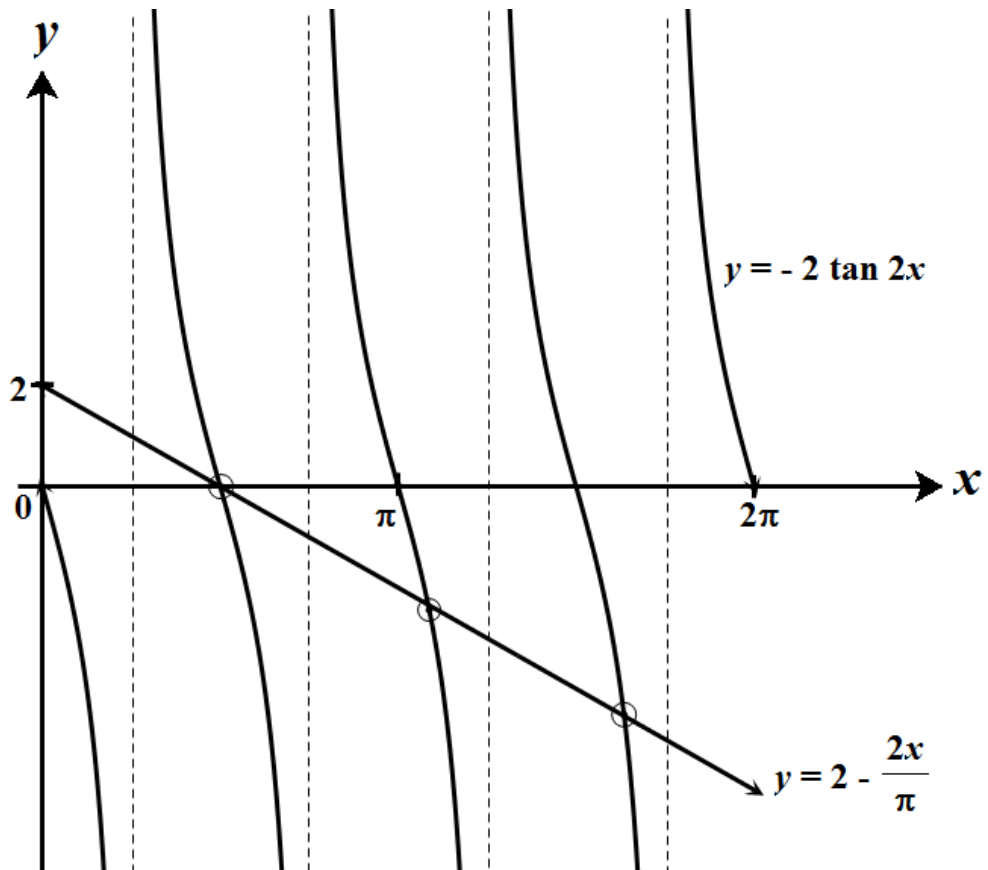
b)



Draw line $y = \frac{x}{2\pi}$, No of solution = 8

7.

a)



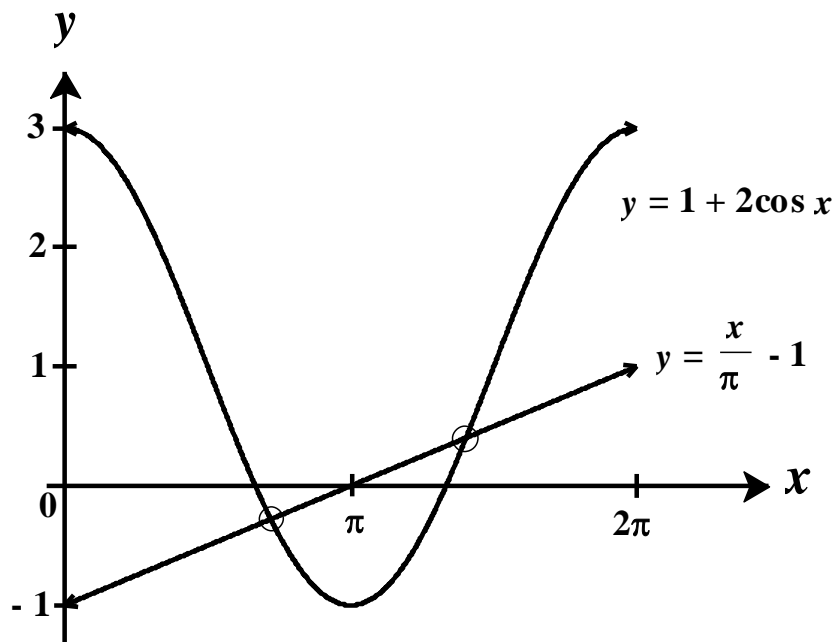
b) Draw line $y = 2 - \frac{2x}{\pi}$, No of solution = 3

8.

a) left side $\frac{2 + 2 \tan^2 \theta}{1 - \tan^2 \theta}$

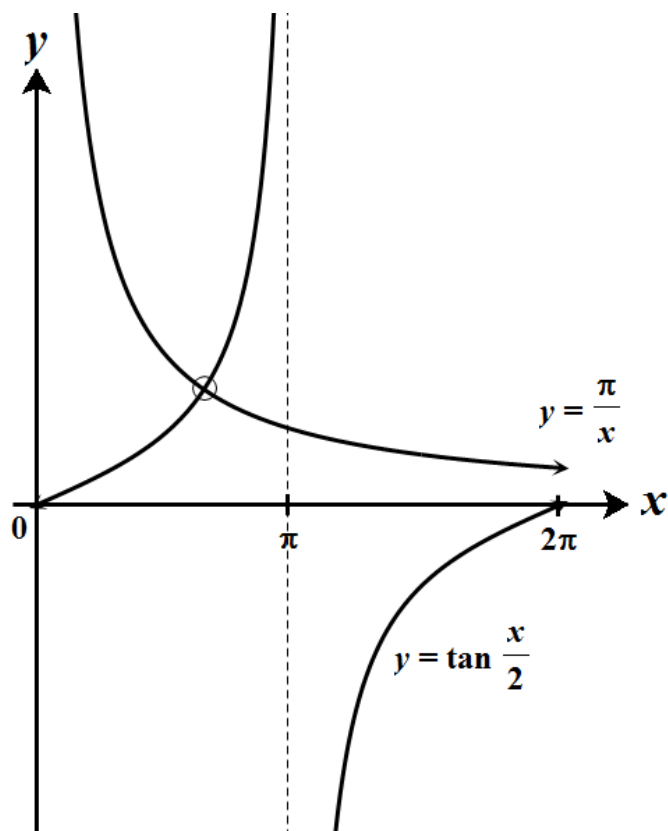
$$\begin{aligned}
 &= \frac{2(1 + \tan^2 \theta)}{1 - \tan^2 \theta} \\
 &= \frac{2\left(\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}\right)}{\left(\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}\right)} \\
 &= \frac{2(1)}{\cos^2 \theta - (1 - \sin^2 \theta)} \\
 &= \frac{2}{\cos 2\theta} \\
 &= 2 \sec \theta
 \end{aligned}$$

b)



c) Draw a line $y = \frac{x}{\pi} - 1$. No of solution = 2.

9. a)

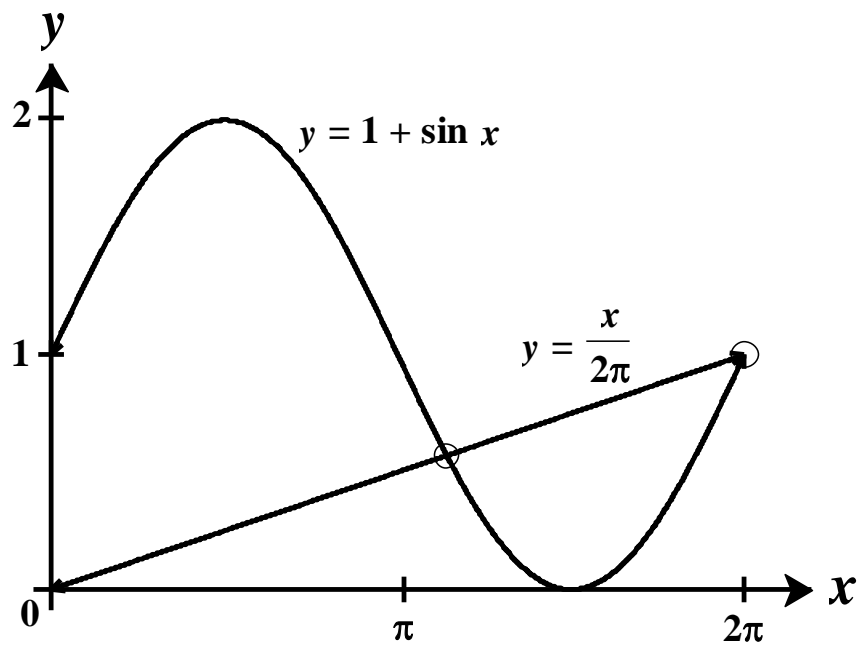


b) c) Draw a line $y = \frac{\pi}{x}$. No of solution = 1.

10.

a) left side $\frac{\cos^2 x}{1 - \sin x}$
 $= \frac{1 - \sin^2 x}{1 - \sin x}$
 $= \frac{(1 - \sin x)(1 + \sin x)}{1 - \sin x}$
 $= 1 + \sin x$

b)



Draw line $y = \frac{x}{2\pi}$. No of solution = 2 .