

**PAPER 1**

1 5 Omega. The standard deviation of the marks of 5 Omega is the lowest among the three classes.

2 (a)  $S = \{(H, H), (H, T), (T, H), (T, T)\}$

(b)  $X = 0, 1, 2$

3 (a) Number of different ways =  $5!$

$$= 120$$

(b) Number of different ways =  $3 \times 3! \times 2$

$$= 3 \times 6 \times 2 = 36$$

4 (a) Probability =  $\left(\frac{3}{7} \times \frac{3}{7}\right) + \left(\frac{4}{7} \times \frac{4}{7}\right) = \frac{25}{49}$

(b) Probability =  $\left(\frac{4}{7} \times \frac{3}{7} \times \frac{3}{7}\right) + \left(\frac{3}{7} \times \frac{4}{7} \times \frac{3}{7}\right)$

$$= \frac{72}{343}$$

5  $\int_1^h (2x - 6)dx = -4$

$$\left[\frac{2x^2}{2} - 6x\right]_1^h = -4$$

$$h^2 - 6h - (1 - 6) + 4 = 0$$

$$h^2 - 6h + 9 = 0$$

$$(h - 3)(h - 3) = 0$$

$$h - 3 = 0$$

$$h = 3$$

$$6 \quad V = 125$$

Let the length of side be  $x$  cm.

$$x^3 = 125 = 5^3$$

$$x = 5$$

$$A = 6x^2$$

$$\frac{dA}{dx} = 12x$$

$$= 12 \times 5 = 60$$

$$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt}$$

$$15 = 60 \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{15}{60} = \frac{1}{4} \text{ cm s}^{-1}$$

$$7 (a) \text{ At } P, y = 0, \quad \frac{2x - 6}{x + 2} = 0$$

$$2x = 6$$

$$x = 3$$

$$P = (3, 0)$$

$$y = \frac{2x - 6}{x + 2}$$

$$\frac{dy}{dx} = \frac{(x + 2) \cdot 2 - (2x - 6) \cdot 1}{(x + 2)^2}$$

$$= \frac{2x + 4 - 2x + 6}{(x + 2)^2}$$

$$= \frac{10}{(x + 2)^2}$$

$$\text{At } P, \frac{dy}{dx} = \frac{10}{(3 + 2)^2} = \frac{2}{5}$$

$$\frac{dy}{dx} = 2q$$

$$\frac{2}{5} = 2q$$

$$q = \frac{1}{5}$$

$$8 \quad (0, 5k) : x = 0, y = 5k$$

$$2y = 3x + h + k$$

$$2(5k) = 0 + h + k$$

$$10k - k = h$$

$$h = 9k$$

$$9 (a) \quad m_1 m_2 = -1 \quad 3 \times q = -1$$

$$q = -\frac{1}{3}$$

$$(b) \quad y = 3x + 4 \quad \dots\dots\dots \textcircled{1}$$

$$y = -\frac{1}{3}x - 6 \quad \dots\dots\dots \textcircled{2}$$

$$3x + 4 = -\frac{1}{3}x - 6$$

$$9x + 12 = -x - 18$$

$$10x = -30$$

$$x = -3$$

$$y = -9 + 4 = -5$$

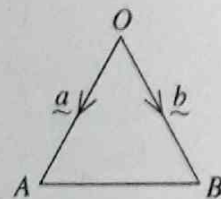
$$F = (-3, -5)$$

$$10 (a) \quad \vec{AC} + \vec{CE} + \vec{CB} = \vec{AE} + \vec{EF} = \vec{AF}$$

$$(b) \quad \vec{AB} = \vec{OB} - \vec{OA}$$

$$= \vec{b} - \vec{a}$$

$$|\vec{AB}| = 3 \text{ units}$$



Unit vector in the direction of  $\vec{AB}$

$$= \frac{\vec{AB}}{|\vec{AB}|}$$

$$= \frac{1}{3}(\vec{b} - \vec{a})$$

$$11 (a) \quad f(x) = x$$

$$3x - 2 = x$$

$$3x - x = 2$$

$$2x = 2$$

$$x = 1$$

$$(b) \quad f(x) = 3x - 2$$

$$f(2 - h) = 3(2 - h) - 2$$

$$= 6 - 3h - 2$$

$$= 4 - 3h$$

$$8 - 3h = 4h$$

$$7h = 4$$

$$h = \frac{4}{7} = 1\frac{1}{7}$$

$$12 \quad m(x) = px + 1, h(x) = 3x - 5$$

$$mh(x) = m(3x - 5)$$

$$= p(3x - 5) + 1$$

$$= 3px - 5p + 1$$

$$\text{Given } mh(x) = 3px + q$$

$$3px - 5p + 1 = 3px + q$$

Comparing both sides of the equation,

$$q = -5p + 1$$

$$5p = 1 - q$$

$$p = \frac{1 - q}{5}$$

$$13 (a) \quad 3x + 1 = y$$

$$3x = y - 1$$

$$x = \frac{y - 1}{3}$$

$$g^{-1}(x) = \frac{x - 1}{3}$$

$$(b) \quad fg(x) = 9x^2 + 6x - 4$$

$$f(3x + 1) = 9x^2 + 6x - 4$$

$$\text{Let } 3x + 1 = u, \text{ then } x = \frac{u - 1}{3}$$

$$f(u) = 9\left(\frac{u - 1}{3}\right)^2 + 6\left(\frac{u - 1}{3}\right) - 4$$

$$= \frac{9(u - 1)^2}{9} + 2(u - 1) - 4$$

$$= u^2 - 2u + 1 + 2u - 2 - 4$$

$$f(u) = u^2 - 5$$

$$f(x) = x^2 - 5$$

$$14 (a) \quad \log_a 49 = \log_a 7^2$$

$$= 2 \log_a 7 = 2r$$

$$(b) \quad \log_7 343 a^2 = \log_7 343 + \log_7 a^2$$

$$= \log_7 7^3 + 2 \log_7 a$$

$$= 3 \log_7 7 + 2\left(\frac{1}{\log_a 7}\right) = 3 + \frac{2}{r}$$

$$15 \quad 3^p = 5^q$$

$$\log 3^p = \log 5^q$$

$$p \log 3 = q \log 5$$

$$\log 3 = \frac{q}{p} \log 5 \quad \dots\dots\dots \textcircled{1}$$

$$5^q = 15^r$$

$$5^q = (3 \times 5)^r$$

$$5^q = 3^r \times 5^r$$

$$\log 5^q = \log (3^r \times 5^r)$$

$$= \log 3^r + \log 5^r$$

$$q \log 5 = r \log 3 + r \log 5 \quad \dots\dots\dots \textcircled{2}$$

Substitute  $\textcircled{1}$  into  $\textcircled{2}$  :

$$q \log 5 = r\left(\frac{q \log 5}{p}\right) + r \log 5$$

$$q = \frac{rq}{p} + r$$

$$pq = rq + rp$$

$$pq = r(q + p)$$

$$r = \frac{pq}{p + q}$$

$$16 \quad y = 2x^2 - \frac{q}{x}$$

$$xy = 2x^3 - q$$

$$Y = mX + c$$

$$m = 2, c = -q$$

$$m = \frac{13 - 3}{h - 0}$$

$$2 = \frac{10}{h}$$

$$2h = 10$$

$$h = 5$$

$$c = 3$$

$$-q = 3$$

$$q = -3$$

$$17 \quad 3x^2 + 8x + 7 = 0$$

$$a = 3, b = 8, c = 7$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{8}{3}$$

$$\alpha\beta = \frac{c}{a} = \frac{7}{3}$$

$$\text{Sum of new roots} = 3\alpha + 3\beta$$

$$= 3(\alpha + \beta)$$

$$= 3\left(-\frac{8}{3}\right)$$

$$= -8$$

$$\text{Product of new roots} = 3\alpha \times 3\beta$$

$$= 9\alpha\beta$$

$$= 9\left(\frac{7}{3}\right)$$

$$= 21$$

$$\text{The equation is } x^2 - (-8)x + 21 = 0$$

$$x^2 + 8x + 21 = 0$$

$$18 \quad f(x) = x^2 + 2wx + 3w - 2$$

$$a = 1, b = 2w, c = 3w - 2$$

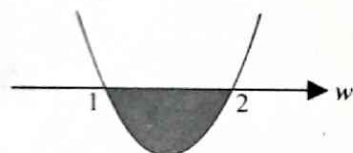
$$b^2 - 4ac < 0$$

$$(2w)^2 - 4(1)(3w - 2) < 0$$

$$4w^2 - 12w + 8 < 0$$

$$w^2 - 3w + 2 < 0$$

$$(w - 1)(w - 2) < 0$$



$$1 < w < 2$$

$$p < w < q$$

$$\text{Thus, } p = 1, q = 2$$

$$19 (a) \quad \text{Length of arc } AB = 16 \text{ cm}$$

$$r \times \theta = 16$$

$$8 \times \theta = 16$$

$$\theta = 2 \text{ radians}$$

$$(b) \quad \theta + x = 2\pi$$

$$x = 2\pi - \theta$$

$$= 2(3.142) - 2 = 4.284 \text{ radians}$$

$$\text{Area of major sector } OAB = \frac{1}{2} r^2 x$$

$$= \frac{1}{2} \times 8^2 \times 4.284$$

$$= 137.088$$

$$= 137.1 \text{ cm}^2$$

$$20$$

$$\tan \alpha = 4 - 3 \cot \alpha$$

$$\tan \alpha = 4 - \frac{3}{\tan \alpha}$$

$$\tan^2 \alpha = 4 \tan \alpha - 3$$

$$\begin{aligned} \tan^2 \alpha - 4 \tan \alpha + 3 &= 0 \\ (\tan \alpha - 1)(\tan \alpha - 3) &= 0 \\ \tan \alpha &= 1 \\ \alpha &= 45^\circ \end{aligned}$$

or  $\tan \alpha = 3$   
 $\alpha = 71^\circ 34'$

Thus,  $\alpha = 45^\circ, 71^\circ 34'$

21 Condensed milk: 70, 65, 60, ...  
 $a = 70, d = -5$   
 $T_n = a + (n - 1)d$   
 $= 70 + (n - 1)(-5)$   
 $= 70 - 5n + 5$   
 $= 75 - 5n \dots\dots\dots \textcircled{1}$

Evaporated milk: 48, 45, 42, ...  
 $a = 48, d = -3$   
 $T_n = a + (n - 1)d$   
 $= 48 + (n - 1)(-3)$   
 $= 48 - 3n + 3$   
 $= 51 - 3n \dots\dots\dots \textcircled{2}$

$\textcircled{1} = \textcircled{2}$ :  $75 - 5n = 51 - 3n$   
 $75 - 51 = 5n - 3n$   
 $2n = 24$   
 $n = 12$

After 12 days, the remainder numbers of cans are the same.

22 (a)  $\frac{2x - 7}{x + 1} = \frac{1}{2}$   
 $2(2x - 7) = x + 1$   
 $4x - 14 = x + 1$   
 $4x - x = 14 + 1$   
 $3x = 15$   
 $x = 5$

(b)  $T_{12} = x + 1$   
 $ar^{11} = 5 + 1$   
 $a \times \left(\frac{1}{2}\right)^{11} = 6$   
 $a \times \frac{1}{2048} = 6$   
 $a = 12288$

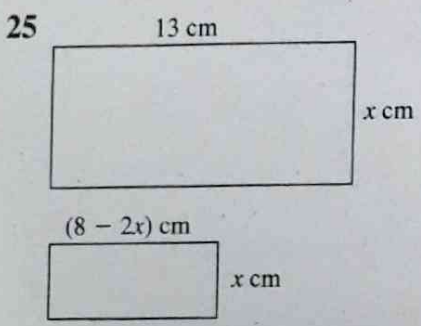
23 4, 4.5, 5.0625, ...  
 $a = 4, r = 1.125, n = 15$   
 $S_n = \frac{a(r^n - 1)}{r - 1}$   
 $S_{15} = \frac{4(1.125^{15} - 1)}{1.125 - 1}$   
 $= 155.3 \text{ minutes}$   
 $= 2.588 \text{ hours}$

Since the time taken is more than 2 hours, Mohan did not qualify.

24 (a)  $n = 3, P(X = r) = {}^3C_r p^r (1 - p)^{3-r}$   
 $P(X = 3) = {}^3C_3 p^3 (1 - p)^0$   
 $\frac{27}{64} = 1 \times p^3 \times 1$   
 $p^3 = \left(\frac{3}{4}\right)^3$   
 $p = \frac{3}{4}$

(b)  $P(\text{lifespan more than 6 months}) = 1 - p$   
 $= 1 - \frac{3}{4}$   
 $= \frac{1}{4}$

Number of bulbs =  $20 \times \frac{1}{4} = 5$



Total area =  $20 \text{ cm}^2$   
 $2(13 \times x) + 2x(8 - 2x) = 20$   
 $26x + 16x - 4x^2 - 20 = 0$   
 $4x^2 - 42x + 20 = 0$   
 $2x^2 - 21x + 10 = 0$   
 $(2x - 1)(x - 10) = 0$   
 $2x - 1 = 0$   
 $x = \frac{1}{2}$

or  $x - 10 = 0$   
 $x = 10$  (rejected)

The width of the wood is  $\frac{1}{2} \text{ cm}$ .

**PAPER 2**

**Section A**

1 (a)

Mass	Number of watermelons (f)	Midpoint x	fx
1.0 - 1.4	6	1.2	7.2
1.5 - 1.9	10	1.7	17
2.0 - 2.4	n	2.2	2.2n
2.5 - 2.9	14	2.7	37.8
3.0 - 3.4	8	3.2	25.6
	n + 38		2.2n + 87.6