PAPER 2

Section A

1 (a)

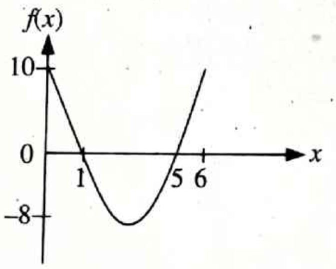
Mass	Number of watermelons (f)	Midpoint x	fx
1.0 – 1.4	6	1.2	7.2
1.5 – 1.9	10	1.7	17
2.0 – 2.4	n	2.2	$2.2n$
2.5 – 2.9	14	2.7	37.8
3.0 – 3.4	8	3.2	25.6
	$n + 38$		$2.2n + 87.6$

$$\text{Mean} = 2.28$$
$$\frac{2.2n + 87.6}{n + 38} = 2.28$$
$$2.2n + 87.6 = 2.28\,(n + 38)$$
$$2.2n + 87.6 = 2.28n + 86.64$$
$$2.28n - 2.2n = 87.6 - 86.64$$
$$0.08n = 0.96$$
$$n = 12$$

(*b*) $L = 1.95,\ N = n + 38$
$$= 12 + 38 = 50$$
$$F = 6 + 10 = 16$$
$$f_m = 12$$
$$c = 2.45 - 1.95 = 0.50$$
$$\text{Median} = 1.95 + \left(\frac{\frac{1}{2}(50) - 16}{12}\right)0.5$$
$$= 1.95 + 0.375$$
$$= 2.325$$

2 (*a*) $x - h = 0$
$$x = h$$
$$h = \frac{1 + 5}{2} = 3$$
$$f(x) = 2k$$
$$f(x) = -8$$
$$2k = -8$$
$$k = -4$$

(*b*)



Minimum point $= (h, 2k)$
$$= (3, -8)$$
When $x = 0$, $y = 10$

When $x = 6$, $y = 10$

(*c*) $f(x) = 2(x - h)^2 + 2k$

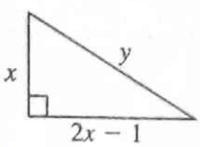
$f(x) = 2(x - 3)^2 - 8$

If the graph is reflected in the x-axis, then the new equation is

$f(x) = -[2(x - 3)^2 - 8]$

$f(x) = -2(x - 3)^2 + 8$

3



$$y^2 = x^2 + (2x - 1)^2$$
$$= x^2 + 4x^2 - 4x + 1$$
$$y^2 = 5x^2 - 4x + 1 \quad \ldots\ldots \text{①}$$

Perimeter = 40
$$y + x + 2x - 1 = 40$$
$$y = 40 + 1 - 3x$$
$$y = 41 - 3x \quad \ldots\ldots \text{②}$$

Substitute ② into ①:
$$(41 - 3x)^2 = 5x^2 - 4x + 1$$
$$1\,681 - 246x + 9x^2 = 5x^2 - 4x + 1$$
$$4x^2 - 242x + 1\,680 = 0$$
$$2x^2 - 121x + 840 = 0$$
$$(2x - 105)(x - 8) = 0$$
$$2x - 105 = 0$$
$$x = \frac{105}{2} = 52\frac{1}{2} \text{ (rejected)}$$

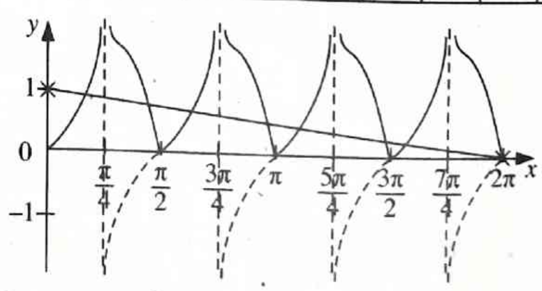
or $x - 8 = 0$
$$x = 8$$
$$y = 41 - 3x$$
$$= 41 - 3(8) = 17$$
$$2x - 1 = 2(8) - 1$$
$$= 15$$

The sides are 8 m, 15 m and 17 m.

4 (*a*) Left-hand side $= \dfrac{\sin 2x}{\tan^2 x + 2\cos^2 x - \sec^2 x}$
$$= \frac{\sin 2x}{\sec^2 x - 1 + 2\cos^2 x - \sec^2 x}$$
$$= \frac{\sin 2x}{2\cos^2 x - 1}$$
$$= \frac{\sin 2x}{\cos 2x}$$
$$= \tan 2x = \text{Right-hand side}$$

(*b*) $y = \tan 2x$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y	0	∞	0	∞	0	∞	0	∞	0



(*c*) $\left|\dfrac{\sin 2x}{\tan^2 x + 2\cos^2 x - \sec^2 x}\right| + \dfrac{x}{2\pi} = 1$
$$|\tan 2x| = 1 - \frac{x}{2\pi}$$
$$y = 1 - \frac{x}{2\pi}$$

x	0	2π
y	1	0

Number of solutions = 8

5 (*a*) Resultant velocity of boat $A = \underset{\sim}{a} + \underset{\sim}{w}$

$$V_A = 2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$$

$$= 3\underset{\sim}{i} + \frac{3}{2}\underset{\sim}{j}$$

$$V_A = 3\left(\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}\right) \text{ .... } ①$$

Resultant velocity of boat $B = \underset{\sim}{b} + \underset{\sim}{w}$

$$V_B = 6\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$$

$$= 7\underset{\sim}{i} + \frac{7}{2}\underset{\sim}{j}$$

$$V_B = 7\left(\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}\right) \text{ .... } ②$$

From ①, $\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j} = \frac{1}{3}V_A$

From ②, $\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j} = \frac{1}{7}V_B$

$$\frac{1}{7}V_B = \frac{1}{3}V_A$$

$$V_B = \frac{7}{3}V_A = 2\frac{1}{3}V_A$$

The resultant velocity of boat B is $2\frac{1}{3}$ times the resultant velocity of boat A.

(*b*) (i) Resultant velocity of boat C

$$= \underset{\sim}{c} + \underset{\sim}{w}$$

$$= 2\underset{\sim}{i} - \frac{3}{2}\underset{\sim}{j} + \underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j} = 3\underset{\sim}{i} - \underset{\sim}{j}$$

(ii) $|3\underset{\sim}{i} - \underset{\sim}{j}| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$ units

Unit vector in the direction of boat C

$$= \frac{3\underset{\sim}{i} - \underset{\sim}{j}}{\sqrt{10}} = \frac{1}{\sqrt{10}}(3\underset{\sim}{i} - \underset{\sim}{j})$$

6 (*a*) Perimeter = 4 m

$$2x + y + y + \frac{1}{2}(2\pi x) = 4$$

$$2x + 2y + \pi x = 4$$

$$2y = 4 - 2x - \pi x$$

$$y = 2 - x - \frac{\pi}{2}x$$

Total surface area

= Area of semicircle + area of rectangle

$$A = \frac{1}{2}\pi x^2 + (2x \times y)$$

$$= \frac{1}{2}\pi x^2 + 2x\left(2 - x - \frac{\pi}{2}x\right)$$

$$= \frac{1}{2}\pi x^2 + 4x - 2x^2 - \pi x^2$$

$$A = 4x - 2x^2 - \frac{1}{2}\pi x^2$$

(*b*) $\frac{dA}{dx} = 4 - 4x^2 - \pi x$

$\frac{dA}{dx} = 0,\quad 4 - 4x - \pi x = 0$

$$4 = 4x + \pi x$$

$$x(4 + \pi) = 4$$

$$x = \frac{4}{4 + \pi} = 0.5601$$

$$\frac{d^2A}{dx^2} = -4 - \pi < 0$$

A is maximum.

Width $= 2x = 2 \times 0.5601 = 1.1202$ m

Section B

7 (*a*) $x + x + 30 = 180$

$$2x = 150$$

$$x = 75$$

$$= \frac{75 \times 3.142}{180} = 1.3092 \text{ radians}$$

$\angle OAB = 1.3092$ radians

(*b*) Length of arc $BC = r\theta$

$$= 8.8 \times 1.3092 = 11.52 \text{ cm}$$

Perimeter of sector $BAC = 11.52 + 8.8 + 8.8$

$$= 29.12 \text{ cm}$$

(*c*) $30° = \frac{30 \times 3.142}{180} = 0.5237$ radian

Area of segment AB

= Area of sector AOB − Area of triangle AOB

$$= \left(\frac{1}{2} \times 17 \times 17 \times 0.5237\right) - \left(\frac{1}{2} \times 17 \times 17 \times \sin 30\right) = 3.4247 \text{ cm}^2$$

Area of sector $BAC = \frac{1}{2} \times 8.8 \times 8.8 \times 1.3092$

$$= 50.692 \text{ cm}^2$$

Area of the shaded region $= 3.4247 + 50.692$

$$= 54.\,1167 \text{ cm}^2$$

8 (*a*) $y = \frac{4}{x^2} = 4x^{-2}$

$$\frac{dy}{dx} = -8x^{-3}$$

$$= -\frac{8}{x^3}$$

$$= -\frac{8}{2^3} = -1$$

Equation of tangent: $y - y_1 = \frac{dy}{dx}(x - x_1)$

$$y - 1 = -1(x - 2)$$

$$y = -x + 2 + 1$$

$$y = -x + 3$$

$m = -1, c = 3$

(*b*) Area of shaded region

$$= \int_2^4 y\, dx - \frac{1}{2}bh$$

$$= \int_2^4 4x^{-2}\, dx - \left(\frac{1}{2} \times 1 \times 1\right)$$

$$= \left[\frac{4x^{-1}}{-1}\right]_2^4 - \frac{1}{2}$$

$$= \left[-\frac{4}{x}\right]_2^4 - \frac{1}{2}$$

$$= -1 - (-2) - \frac{1}{2}$$

$$= -1 + 2 - \frac{1}{2}$$

$$= \frac{1}{2} \text{ unit}^2$$

(*c*) Volume generated $= \dfrac{38\pi}{81}$

$$\int_2^k \pi y^2\, dx = \frac{38\pi}{81}$$

$$\int_2^k \left(\frac{4}{x^2}\right)^2 dx = \frac{38}{81}$$

$$\int_2^k 16x^{-4}\, dx = \frac{38}{81}$$

$$\left[\frac{16x^{-3}}{-3}\right]_2^k = \frac{38}{81}$$

$$\left[-\frac{16}{3x^3}\right]_2^k = \frac{38}{81}$$

$$-\frac{16}{3k^3} - \left(-\frac{16}{24}\right) = \frac{38}{81}$$

$$\frac{16}{3k^3} = \frac{16}{24} - \frac{38}{81}$$

$$\frac{16}{3k^3} = \frac{16}{81}$$

$$3k^3 = 81$$

$$k^3 = 27$$

$$= 3^3$$

$$k = 3$$

9 (*a*) $n = 8, p = 20\% = 0.2$

$P(X = r) = {}^8C_r\,(0.2)^r\,(0.8)^{8-r}$

$P(X = 3) = {}^8C_3\,(0.2)^3\,(0.8)^5$

$= 0.1468$

(*b*) $\mu = 2, \sigma = m$

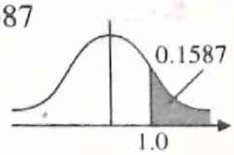
(i) $P(X > 2.5) = 15.87\%$

$$P\left(Z > \frac{2.5 - 2}{m}\right) = 0.1587$$

$$P(Z > 1.0) = 0.1587$$

$$\frac{2.5 - 2}{m} = 1.0$$

$$m = 0.5$$



(ii) $p = P(1.0 < x < 2.5)$

$$= P\left(\frac{1.0 - 2}{0.5} < Z < \frac{2.5 - 2}{0.5}\right)$$

$$= P(-2 < Z < 1)$$

$$= 1 - P(Z < -2) - P(Z > 1)$$

$$= 1 - P(Z > 2) - P(Z > 1)$$

$$= 1 - 0.0228 - 0.1587$$

$$= 0.8185$$

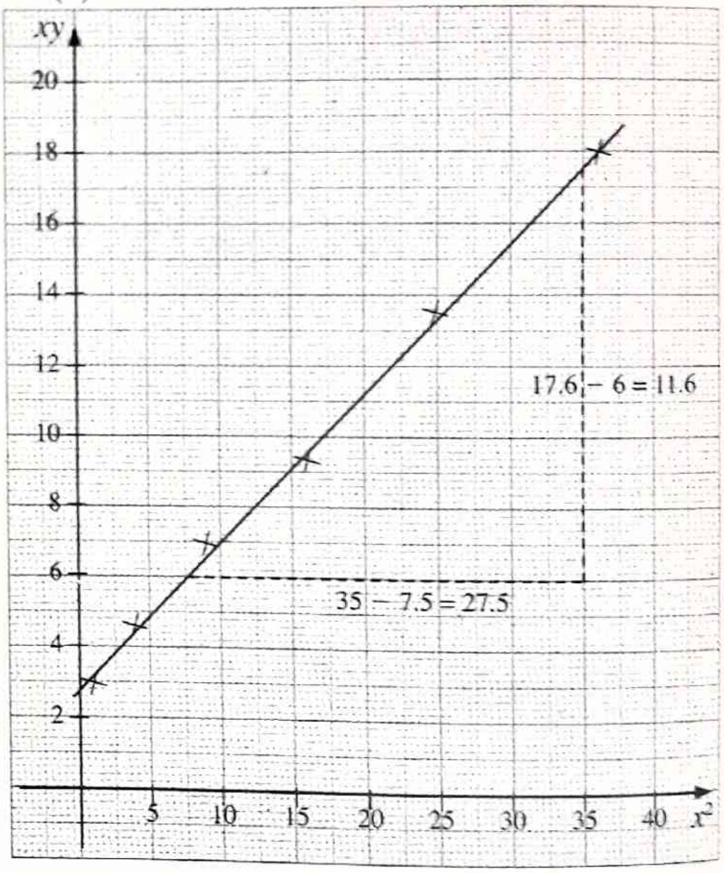
Number of pineapples $= 0.8185 \times 1\,320$

$= 1\,080$

10 (*a*)

x	1	2	3	4	5	6
y	3.10	2.30	2.33	2.35	2.72	3.00
x^2	1	4	9	16	25	36
xy	3.10	4.60	6.99	9.40	13.60	18.00

(*b*)



(*c*) $y = 2px + \dfrac{q}{5x}$

$$xy = 2px^2 + \frac{q}{5}$$

$$Y = mX + C$$

(i) $m = \dfrac{11.6}{27.5}$

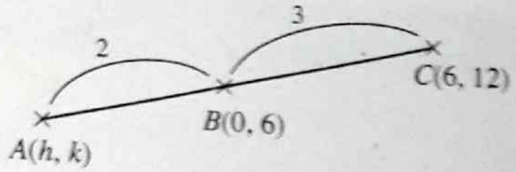
$$2p = \frac{11.6}{27.5}$$

$$p = \frac{1}{2} \times \frac{11.6}{27.5} = 0.211$$

(ii) $C = 3 \qquad \dfrac{q}{5} = 3$

$$q = 15$$

11 (*a*) (i)



$$B = \left(\frac{2(6) + 3h}{2 + 3}, \frac{2(12) + 3k}{2 + 3}\right)$$

$$(0, 6) = \left(\frac{12 + 3h}{5}, \frac{24 + 3k}{5}\right)$$

$$\frac{12 + 3h}{5} = 0 \qquad \frac{24 + 3k}{5} = 6$$

$$3h = -12 \qquad 3k = 6$$

$$h = -4 \qquad k = 2$$

$A = (-4, 2)$

(ii) $m_{AD} = \dfrac{2 - (-6)}{-4 - 2}$

$$= -\frac{8}{6} = -\frac{4}{3}$$

Equation of AD:

$$y - y_1 = m_{AD}(x - x_1)$$

$$y - (-6) = -\frac{4}{3}(x - 2)$$

$$y + 6 = -\frac{4}{3}x + \frac{8}{3}$$

$$3y + 18 = -4x + 8$$

$$3y = -4x - 10$$

(iii) Area of ACD

$$= \frac{1}{2}\begin{vmatrix} -4 & 2 & 6 & -4 \\ 2 & -6 & 12 & 2 \end{vmatrix}$$

$$= \frac{1}{2}\left|(24 + 24 + 12) - (4 - 36 - 48)\right|$$

$$= \frac{1}{2}\left|60 - (-80)\right| = 70 \text{ unit}^2$$

(*b*) Let $P = (x, y)$,

$$PC = 2PD$$

$$\sqrt{(x - 6)^2 + (y - 12)^2} = 2\sqrt{(x - 2)^2 + (y + 6)^2}$$

$$4\left[(x - 2)^2 + (y + 6)^2\right] = (x - 6)^2 + (y - 12)^2$$

$$4(x^2 - 4x + 4 + y^2 + 12y + 36) =$$

$$x^2 - 12x + 36 + y^2 - 24y + 144$$

$$4x^2 + 4y^2 - 16x + 48y + 160 =$$

$$x^2 + y^2 - 12x - 24y + 180$$

$$3x^2 + 3y^2 - 4x + 72y - 20 = 0$$

Section C

12 (*a*) $\dfrac{P_{11}}{P_{08}} \times 100 = 140$

$$\frac{70}{P_{08}} \times 100 = 140$$

$$P_{08} = \frac{70 \times 100}{140} = 50$$

(*b*)

Raw material	Price index in 2015 based on 2008
A	$140 \times \frac{115}{100} = 161$
B	$120 \times \frac{105}{100} = 126$
C	$160 \times \frac{100}{100} = 160$
D	$150 \times \frac{90}{100} = 135$

(*c*) (i) Composite index

$$= \frac{(161 \times 30) + (126 \times 10) + (160 \times 20) + (135 \times 25)}{30 + 10 + 20 + 25}$$

$$= \frac{12\,665}{85} = 149$$

(ii) $\dfrac{P_{15}}{P_{08}} \times 100 = 149$

$$\frac{268.20}{P_{08}} \times 100 = 149$$

$$P_{08} = \frac{268.20 \times 100}{149}$$

$$= \text{RM}180$$

13 (*a*) (i) $40x + 20y \leq 2\,000$

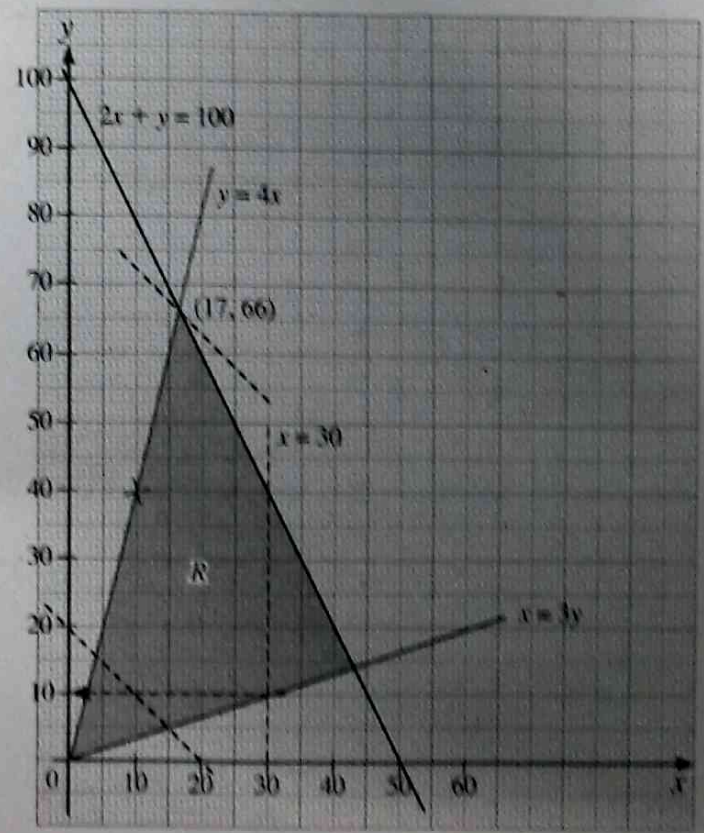
$$2x + y \leq 100$$

(ii) $y \leq 4x$

(*b*) $3y \geq x$

The number of hacksaws is not more than 3 times the number of chisels.

(*c*)



(*d*) (i) When $x = 30$, y minimum $= 10$
The minimum number of chisels is 10.

(ii) Total number $= x + y$

$x + y = 20$

Maximum point $= (17, 66)$

Maximum total number $= x + y$
$= 17 + 66$
$= 83$

14 (*a*) $v = pt^2 - 6t$

$a = \frac{dv}{dt} = 2pt - 6$

When $t = 3$, $a = 18$ $\quad 18 = 2p(3) - 6$

$6p = 24$

$p = 4$

(*b*) $a < 0$, $\quad 2pt - 6 = 0$

$8t - 6 < 0$

$8t < 6$

$t < \frac{6}{8}$

$0 \leqslant t < \frac{3}{4}$

(*c*) $v = 0$, $pt^2 - 6t = 0$

$4t^2 - 6t = 0$

$2t(2t - 3) = 0$

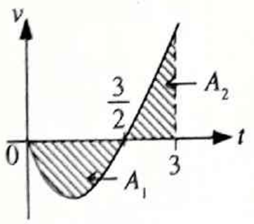
$2t = 0$

$t = 0$

or $2t - 3 = 0$

$t = \frac{3}{2}$

(*d*)



$A_1 = \int_0^{\frac{3}{2}} v\,dt$

$= \int_0^{\frac{3}{2}} 4t^2 - 6t\,dt$

$= \left[\frac{4t^2}{3} - \frac{6t^2}{2}\right]_3^{\frac{3}{2}}$

$= \frac{4}{3}\left(\frac{27}{8}\right) - 3\left(\frac{9}{4}\right) - 0$

$= 4\frac{1}{2} - 6\frac{3}{4}$

$= -2\frac{1}{4}$

$A_1 = 2\frac{1}{4}\ m$

$A_2 = \int_{\frac{3}{2}}^{3} v\,dt$

$= \left[\frac{4}{3}t^3 - 3t^2\right]_{\frac{3}{2}}^{3}$

$= \frac{4}{3}(27) - 27 - \left(\frac{9}{2} - \frac{27}{4}\right)$

$= 9 - \frac{9}{2} + \frac{27}{4}$

$= 11\frac{1}{4}$ m

Total distance travelled $= A_1 + A_2$
$= 2\frac{1}{4} + 11\frac{1}{4}$
$= 13.5$ m

15 (*a*) (i) $AC^2 = 7^2 + 8^2 - 2(7)(8)\cos 80$

$AC = 9.672$ cm

(ii) $\angle ADC + 80 = 180$

$\angle ADC = 100$

$\frac{\text{Sin } \angle CAD}{3} = \frac{\text{Sin } 100}{9.672}$

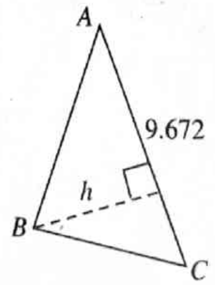
$\text{Sin } \angle CAD = \frac{3 \times \text{Sin } 100}{9.672}$

$\angle CAD = 17°47'$

$\angle ACD = 180° - 100° - 17°47'$
$= 62°13'$

(*b*) (i) Area of $\Delta ABC = \frac{1}{2} \times 7 \times 8 \times \text{Sin } 80$
$= 27.57\ \text{cm}^2$

(ii)



Area of $\Delta ABC = 27.57$

$\frac{1}{2} \times 9.672 \times h = 27.57$

$4.836h = 27.57$

$h = \frac{27.57}{4.836}$

$= 5.701$ cm