

PAPER 1

- 1 (a) B and C  
(b) A  
(c) C

2 (a)  $\int_{-1}^5 f(x) dx = 4$

$\therefore a = -1, b = 5$

(b)  $\int_{-1}^5 f(x) dx + \left| \int_5^9 f(x) dx \right| = 12$

$4 + \left| \int_5^9 f(x) dx \right| = 12$

$\left| \int_5^9 f(x) dx \right| = 8$

$\int_5^9 f(x) dx = -8$

3 (a)  $|\vec{BA}| = \sqrt{3^2 + 4^2} = 5$  units

(b) (i)  $\vec{BC} = \vec{BA} + \vec{AC} = -\underline{b} + \underline{c} = \underline{c} - \underline{b}$

(ii)  $\vec{AD} = \vec{AB} + \vec{BD}$   
 $= \underline{b} + 2\vec{BC}$   
 $= \underline{b} + 2(\underline{c} - \underline{b}) = 2\underline{c} - \underline{b}$

4  $\underline{p} = m\underline{q}$

$\binom{3}{4} = m \binom{k-1}{2}$

$\binom{3}{4} = \binom{mk-m}{2m}$

From the above equation,  $mk - m = 3$  ..... ①

$2m = 4$  ..... ②

From ②:  $2m = 4$

$m = 2$

Substitute  $m = 2$  into ①:  $2k - 2 = 3$

$2k = 3 + 2$

$2k = 5$

$k = \frac{5}{2}$

5  $\frac{25^{h+3}}{125^{p-1}} = 1$

$25^{h+3} = 125^{p-1}$

$(5^2)^{h+3} = (5^3)^{p-1}$

$5^{2h+6} = 5^{3p-3}$

$2h + 6 = 3p - 3$

$3p = 2h + 9$

$p = \frac{2h + 9}{3}$

6  $\log_n 324 - \log_n 2m = 2$

$\log_n 324 - \frac{\log_n 2m}{\log_n m^{\frac{1}{2}}} = 2$

$\log_n 324 - 2 \log_n 2m = \log_n m^2$

$\log_n 324 - \log_n (2m)^2 = \log_n m^2$

$\log_n \frac{324}{4m^2} = \log_n m^2$

$\frac{324}{4m^2} = m^2$

$4m^4 = 324$

$m^4 = 81 = (\pm 3)^4$

$m = \pm 3$  (-3 is rejected)

Thus,  $m = 3$ .

7 (a)  $k = 0, k = 1, k = -1$

(Any one of these answers.)

(b)  $T_n = \frac{3}{2} r^{n-1}$

$T_1 = \frac{3}{2} r^{1-1}$

$= \frac{3}{2} r^0 = \frac{3}{2}$

8  $S_n = \frac{n}{2} [13 - 3n]$

$S_{n-1} = \frac{n-1}{2} [13 - 3(n-1)]$

$= \frac{1}{2} (n-1)(16 - 3n)$

$T_n = S_n - S_{n-1}$

$= \frac{n}{2} (13 - 3n) - \frac{1}{2} (n-1)(16 - 3n)$

$= \frac{13}{2} n - \frac{3}{2} n^2 - \frac{1}{2} (16n - 3n^2 - 16 + 3n)$

$= \frac{13}{2} n - \frac{3}{2} n^2 - \frac{19}{2} n + \frac{3}{2} n^2 + 8$

$= 8 - 3n$

9 (a) 4

(b)  $f(x) = |1 - 2x|$

$f(3) = |1 - 2(3)|$

$= |1 - 6|$

$= |-5|$

$= 5$

(c) When  $x = -2, f(x) = 5$ .

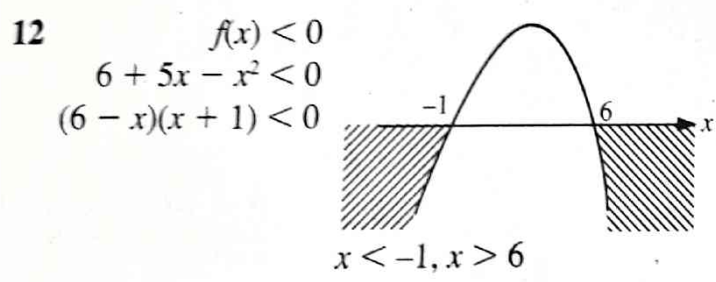
When  $x = 3, f(x) = 5$ .

Domain:  $-2 \leq x \leq 3$

10 (a) Let  $y = g(x)$   
 $= 2x - 8$   
 $2x - 8 = y$   
 $2x = y + 8$   
 $x = \frac{y + 8}{2}$   
 Thus,  $g^{-1}(x) = \frac{x + 8}{2}$

(b)  $g(x) = 2x - 8$   
 $g^2(x) = g[g(x)]$   
 $= g(2x - 8)$   
 $= 2(2x - 8) - 8$   
 $= 4x - 16 - 8$   
 $= 4x - 24$   
 $g^2\left(\frac{3p}{2}\right) = 4\left(\frac{3p}{2}\right) - 24 = 30$   
 $6p - 24 = 30$   
 $6p = 54$   
 $p = 9$

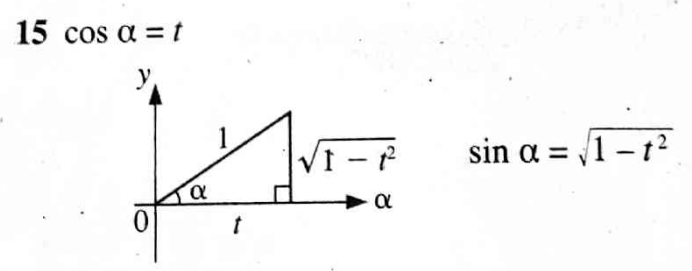
11 (a)  $f(x) = x^2 + 4x + h$   
 $= x^2 + 4x + (2)^2 - (2)^2 + h$   
 $= (x + 2)^2 - 4 + h$   
 (b) Minimum value = 8  
 $-4 + h = 8$   
 $h = 8 + 4 = 12$



13 (a)  $x^2 + (p + 3)x - p^2 = 0$   
 $a = 1, b = p + 3, c = -p^2$   
 Root 1 =  $\alpha$ , Root 2 =  $-\alpha$   
 Sum of the roots =  $-\frac{b}{a}$   
 $\alpha + (-\alpha) = -\frac{(p + 3)}{1}$   
 $-(p + 3) = 0$   
 $p + 3 = 0$   
 $p = -3$   
 Product of roots =  $\frac{c}{a}$   
 $= \frac{-p^2}{1}$   
 $= -(-3)^2 = -9$

(b)  $mx^2 - 5nx + 4m = 0$   
 $a = m, b = -5n, c = 4m$   
 $b^2 = 4ac$   
 $(-5n)^2 = 4m(4m)$   
 $25n^2 = 16m^2$   
 $\frac{m^2}{n^2} = \frac{25}{16}$   
 $\left(\frac{m}{n}\right)^2 = \left(\frac{5}{4}\right)^2$   
 $m : n = 5 : 4$

14 (a) (i) When  $x = 0, y = 2$   
 $m \cos 0 - 1 = 2$   
 $m(1) - 1 = 2$   
 $m = 3$   
 (ii) When  $x = \pi, y = 2$   
 $3 \cos p\pi - 1 = 2$   
 $3 \cos p\pi = 3$   
 $\cos p\pi = 1$   
 $\cos 2\pi = 1$   
 $\therefore p = 2$   
 (b)  $m \cos px = -3$   
 $m \cos px - 1 = -3 - 1$   
 $y = -4$   
 Number of solutions = 1



(a)  $\sin(180^\circ + \alpha) = \sin 180 \cos \alpha + \cos 180 \sin \alpha$   
 $= 0 - \sin \alpha$   
 $= -\sin \alpha$   
 $= -\sqrt{1 - t^2}$   
 (b)  $\sec 2\alpha = \frac{1}{\cos 2\alpha}$   
 $= \frac{1}{2 \cos^2 \alpha - 1} = \frac{1}{2t^2 - 1}$

16 Length of major arc  $AOD = 2r \times 7\alpha = 14r\alpha$   
 Length of minor arc  $BOC = r \times 2\alpha = 2r\alpha$   
 Perimeter of whole diagram = 50  
 $14r\alpha + 2r\alpha + r + r = 50$   
 $16r\alpha + 2r = 50$   
 $8r\alpha + r = 25$   
 $r(8\alpha + 1) = 25$   
 $r = \frac{25}{8\alpha + 1}$

$$17 \int \frac{5}{(2x+3)^n} dx = \int 5(2x+3)^{-n} dx$$

$$= \frac{5(2x+3)^{-n+1}}{(-n+1) \times 2} + c$$

$$= \frac{5}{2(1-n)} \times \frac{1}{(2x+3)^{n-1}} + c$$

$$= \frac{5}{2(1-n)(2x+3)^{n-1}} + c$$

Compare  $\frac{5}{2(1-n)(2x+3)^{n-1}}$  with  $\frac{p}{(2x+3)^5}$ :

$$n-1=5$$

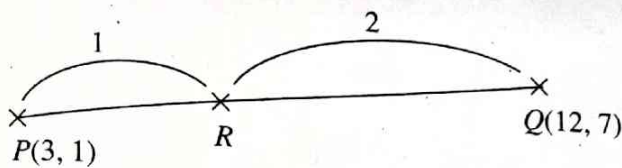
$$n=6$$

$$\frac{5}{2(1-n)} = p$$

$$\frac{5}{2(-5)} = p$$

$$p = -\frac{1}{2}$$

18



$$2PQ = 3RQ$$

$$\frac{PQ}{RQ} = \frac{3}{2}$$

$$R = \left( \frac{1(12) + 2(3)}{1+2}, \frac{1(7) + 2(1)}{1+2} \right)$$

$$= \left( \frac{18}{3}, \frac{9}{3} \right) = (6, 3)$$

19  $y = x + \frac{r}{x^2}$

$$(y-x) = r \left( \frac{1}{x^2} \right) + 0$$

$$Y = mX + C$$

$$m = r, c = 0$$

$$\frac{5p-0}{\frac{h}{2}-0} = r$$

$$5p = \frac{1}{2} hr$$

$$hr = 10p$$

$$h = \frac{10p}{r}$$

20 (a)  $4 + 10 + x + 8 + 7 = 40$

$$x + 29 = 40$$

$$x = 11$$

$$\text{Modal class} = 40 - 59$$

(b) The top ten placings are  $T_{31}, T_{32}, T_{33}, \dots, T_{40}$

$$T_{31} = 59.5 + \frac{5}{8}(79.5 - 59.5)$$

$$= 59.5 + 12.5 = 72$$

A student has to achieve a minimum mark of 72.

Erica qualifies for the reward because her marks  $> 72$  marks.

21  $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$$x + x + x + \frac{1}{16} + x + x = 1$$

$$5x = \frac{15}{16}$$

$$x = \frac{3}{16}$$

$$P(\text{Same numbers}) = P(1, 1) + P(2, 2) + P(3, 3) + P(4, 4) + P(5, 5) + P(6, 6)$$

$$= (x \times x) + (x \times x) + (x \times x) + \left( \frac{1}{16} \times \frac{1}{16} \right) + (x \times x) + (x \times x)$$

$$= 5x^2 + \frac{1}{256}$$

$$= 5 \left( \frac{3}{16} \right)^2 + \frac{1}{256}$$

$$= \frac{23}{128}$$

$$P(\text{Two different numbers}) = 1 - \frac{23}{128} = \frac{105}{128}$$

22 (a) Number of different ways =  ${}^{14}C_3 = 364$

(b)  $(BR), (A), (C), (D), (E)$

Number of ways (Blue cup and red cup are next to each other) =  $5! \times 2! = 240$

$$\text{Number of different ways} = 6! - 240 = 720 - 240 = 480$$

23  $\sum x = 2 + 3 + 4 + 5 + 6 = 20$

$$\sum x^2 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 90$$

$$\text{Mean} = \frac{20}{5} = 4$$

$$\text{Variance} = \frac{\sum x^2}{n} - (\bar{x})^2$$

$$= \frac{90}{5} - 4^2 = 2$$

$$\text{New mean} = 17$$

$$4m + n = 17 \dots\dots\dots \textcircled{1}$$

$$\text{New standard deviation} = 4.242$$

$$m\sqrt{2} = 4.242$$

$$m = \frac{4.242}{\sqrt{2}} = 2.9995 \approx 3$$

Substitute  $m = 3$  into  $\textcircled{1}$ :  $4(3) + n = 17$   
 $n = 5$



$$24 \quad (a) \quad P(X=0) + P(X=1) + P(X=2) + P(X=3) = 1$$

$$P(X=0) + a + b + P(X=3) = 1$$

$$P(X=0) + P(X=3) = 1 - a - b$$

$$P(X=0) + P(X > 2) = 1 - a - b$$

$$(b) \quad P(X=0) = \frac{27}{343}$$

$${}^3C_0 (p^0) (1-p)^3 = \frac{27}{343}$$

$$1 \times 1 \times (1-p)^3 = \left(\frac{3}{7}\right)^3$$

$$1-p = \frac{3}{7}$$

$$p = \frac{4}{7}$$

$$25 \quad (a) \quad P(X < h) = 0.5 - 0.2881$$

$$P(X < h) = 0.2119$$

$$P(X < -0.8) = 0.2119$$

$$h = -0.8$$

$$(b) \quad X = 58.8$$

$$\frac{X - \mu}{\sigma} = \frac{58.8 - \mu}{\sigma}$$

$$Z = \frac{58.8 - \mu}{4}$$

$$h = \frac{58.8 - \mu}{4}$$

$$-0.8 = \frac{58.8 - \mu}{4}$$

$$-3.2 = 58.8 - \mu$$

$$\mu = 58.8 + 3.2 = 62$$